SOLUTION PROBLEM OF THE MONTH, NOVEMBER 2017

Let $\mathcal{M}(0,1,2)$ be the set of rectangular matrices whose entries are 0, 1, or 2.

A matrix in $\mathcal{M}(0,1,2)$ is said to be *defective* if there exists a 2×2 submatrix all whose entries are equal to each other. A matrix with no defect is called *perfect*.

Find a 10×10 *perfect matrix in* $\mathcal{M}(0,1,2)$.

Solution. Here is one such possible construction.

$$M=\begin{bmatrix}
0 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 1 & 2\\
0 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2\\
1 & 0 & 2 & 2 & 0 & 1 & 2 & 0 & 0 & 1\\
1 & 1 & 0 & 2 & 2 & 0 & 1 & 0 & 2 & 0\\
2 & 1 & 1 & 0 & 2 & 2 & 0 & 0 & 1 & 1\\
2 & 2 & 2 & 0 & 1 & 0 & 1 & 1 & 0 & 1\\
1 & 2 & 1 & 0 & 0 & 2 & 2 & 1 & 2 & 0\\
2 & 1 & 0 & 2 & 1 & 1 & 0 & 2 & 0 & 2\\
1 & 0 & 2 & 1 & 1 & 2 & 0 & 2 & 1 & 0\\
0 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 0 & 0
\end{bmatrix}$$