An infinite sequence of quadruples begins with the five quadruples (1, 3, 8, 120), (2, 4, 12, 420), (3, 5, 16, 1008), (4, 6, 20, 1980), (5, 7, 24, 3432). Each quadruple \((a, b, c, d)\) in this sequence has the property that the six numbers \(ab + 1\), \(ac + 1\), \(bc + 1\), \(ad + 1\), \(bd + 1\), and \(cd + 1\) are all perfect squares. Derive a formula for the \(n\)-th quadruple in the sequence and demonstrate that the property holds for every quadruple generated by the formula.

Submit your solutions to professor Dan Ismailescu, Mathematics Department via email at dan.p.ismailescu@hofstra.edu, or bring it in person at 103A Roosevelt Hall.