Mathematics KSBs

Lego Robotics

Developed by Brian Schor

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Linear Movement Based on Power Level Setting

To better understand the design challenge, you will first need to gather information in order to build a knowledge base.

Knowledge and Skill Builder A: Investigating Distance versus Time and Power Level Relationships

In order to determine how rate of speed – ***velocity*** – that the two motors will make the robotic controlled motor vehicle travel in a straight line it is necessary to determine the distance that the motor vehicle will travel linearly within a given set time frame and set power level. For this activity, both the left and right vehicle's wheel motors must be programmed to operate in a "forward" direction and set to the exact same power levels. Several experimental trials should then be conducted whereby the timing mechanism for both motors' forward operation should then terminated after each of the following set amounts of time – one second, two seconds, three seconds, four seconds, five seconds, etc … until 10 seconds – before stopping. This should be done for each of the five power level setting. First, the investigator must mark the exact starting point for the motor vehicle; then, once the program is terminated, the investigator must measure the entire straight lined – ***linear*** – distance that the motor vehicle travels (in centimeters) by noting the motor vehicle's stopping location.

This information can be entered into a table such as the following:

Left & Right Motor Power Levels For Wheels	Power Level = 1	Power Level = 2	Power Level = 3	Power Level = 4	Power Level = 5
Distance After 1 Second					
Distance After 2 Seconds					
Distance After 3 Seconds					
Distance After 4 Seconds					
Distance After 5 Seconds					
Distance After 6 Seconds					
Distance After 7 Seconds					
Distance After 8 Seconds					
Distance After 9 Seconds					
Distance After 10 Seconds					

<u>Knowledge and Skill Builder B</u>: Graphing the Distance versus Time and Power Level Relationships

To visualize the distance versus time and power level relationship better, a ***line graph*** is most appropriate inasmuch as it shows ***trends*** over time. To construct this type of graph:

1. Draw a horizontal and vertical ***axis***.
2. Five distinct lines will be needed – one to represent each of the five respective power levels; so, colored pencils or markers are appropriate. Each color should correspond to a specific power level and must be entered into the ***key*** for the graph.
3. ***Label*** the horizontal axis as "Time in Seconds"; and, label the vertical axis as "Distance in Centimeters". Include a ***title*** of the graph.
   a. Since the ***range*** of times examined was from 0 to 10 seconds, the vertical lines intersecting the horizontal axis should also be labeled from zero to ten.
4. Determine the ***maximum*** distance that the motor vehicle traveled should occur when it was set to the highest power level (5) and let run for the longest period of time (10 seconds), the vertical axis should range from zero to a distance slightly greater than this maximum distance; and, the **interval** must be a ***constant*** – equal to the highest value divided by the number of horizontal lines (not including the horizontal axis).
5. Graph (***plot***) the data – the five distinct lines.
6. ***Connect*** the points for each line.

Using the data from **<u>Knowledge and Skill Builder A</u>**, the line graph representing the distance versus time and power level relationships can be plotted on the next page.

Did each of the five motor power levels make the motor vehicle travel at a constant – ***linear*** – rate of speed? Or, was the rate of speed more like a ***quadratic*** function? How do you know? Justify your explanation using aspects from the line graph on the next page.

___

___

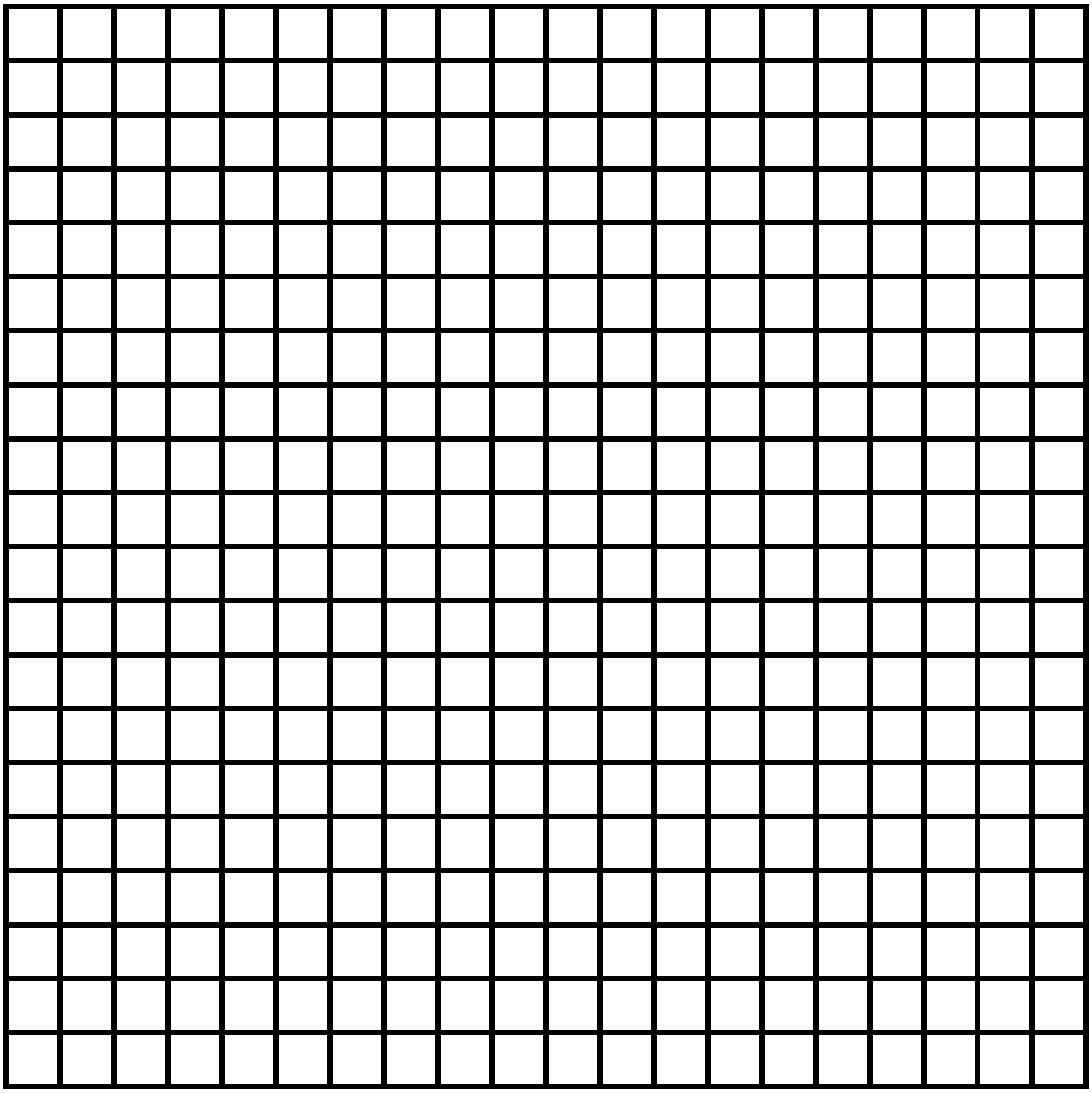
Where was the acceleration, for each of the five motor power levels, of the motor vehicle the greatest? Where was the acceleration the least? Or, was the acceleration constant over time? Justify your explanation using aspects from the line graph on the next page.

___

___

Line Graph of Distance versus Time and Power Level

D I S T A N C E I N C E N T I M E T E R S



0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0

TIME IN SECONDS

KEY

= Power Level 1
= Power Level 2
= Power Level 3
= Power Level 4
= Power Level 5

Knowledge and Skill Builder C: Equations of Distance versus Time for each Power Level

In order to determine the programming time that the wheels' motors should be set to operate in future tasks, a specific ***equation*** can be developed and written for the mathematical relationship between the linear distance (in centimeters) that the motor vehicle traveled versus the time (in seconds) that the motors attached to the left and right wheels are programmed to operate, for each of the five power level settings. Since each individual power level should operate at a constant rate of speed – discounting friction and the weakening of the batteries – the most appropriate equation is a ***linear relationship*** between distance and time.

The ***linear equations***, for each of the five power levels, have the following properties:

1. Since motor vehicle starts at has a starting distance of 0, the ***y-intercept*** is also zero.
2. The ***slope*** – rate of speed – is the measured ratio of the distance traveled (in centimeters) divided by time (in seconds).
3. The equation becomes $\mathbf{D = R_P \cdot time}$, where **D** represents the distance traveled and $\mathbf{R_P}$ represents the ***velocity*** (rate of speed) for a given of power level, **p = {1, 2, 3, 4, 5}**.

Using the data from **Knowledge and Skill Builder A**, the equation for the mathematic relationship between distance (in centimeters) and time (in seconds), for each power level, can be formed using the following table.

Motor Power Levels for the Left and Right Wheels	Velocity – Rate of Speed (Centimeters per Second)	Equation of Distance Versus Time
Power Level = 1	$\mathbf{R_1}$ = ___________	$\mathbf{D = R_1 \cdot time}$ **D** = ___**• time**
Power Level = 2	$\mathbf{R_2}$ = ___________	$\mathbf{D = R_2 \cdot time}$ **D** = ___**• time**
Power Level = 3	$\mathbf{R_3}$ = ___________	$\mathbf{D = R_3 \cdot time}$ **D** = ___**• time**
Power Level = 4	$\mathbf{R_4}$ = ___________	$\mathbf{D = R_4 \cdot time}$ **D** = ___**• time**
Power Level = 5	$\mathbf{R_5}$ = ___________	$\mathbf{D = R_5 \cdot time}$ **D** = ___**• time**

Why might ***inaccuracies*** in experimental measurements – either the measured distance or recorded time – affect future predictions resulting from these equations? Explain your analysis.

How do ***friction***, ***inertia***, and the effect of ***fading batteries*** affect future predictions resulting from these equations – which are based on motor power levels and the speed that the motor turns? Explain your analysis.

Is the ***velocity*** – rate of speed – for the motor vehicle traveling backwards the same as the ***velocity*** traveling forward for the motor vehicle? Prove your findings through experimentation for each of the five motor speeds.

Why is it important that programming make the motor vehicle travel the exact same distance forward or backwards each time the program is run, for any given motor speed? Explain your analysis.

Is distance traveled using time and power speed an accurate measurement? Explain your analysis.

Knowledge and Skill Builder D: Deriving the Equation for Predicting Time Based On Power Levels

In order to predict the programming time in future tasks that the wheels' motors should be set to operate in order to travel a given distance for a specific power level, the equations of distance (in centimeters) versus time (in seconds) for each power level need to be analyzed.

The equations of distance versus time, for each of the five power levels, can be represented by

$\mathbf{D = R_p \cdot time,}$	where $\mathbf{R_p}$ represents velocity for the given power level, $\mathbf{p = \{1, 2, 3, 4, 5\}}$

These were velocities, $\mathbf{R_p}$ were derived in **Knowledge and Skill Builder C**.

Given the equation "$\mathbf{D = R_p \cdot time}$", the variable "***time***" can be solved for in terms of the other two variables, **D** and $\mathbf{R_p}$, to yield the resulting transformed equation:

$\mathbf{Time = \frac{D}{R_p}}$	where $\mathbf{R_p}$ represents velocity for the given power level, $\mathbf{p = \{1, 2, 3, 4, 5\}}$

How closely does this formula predict the distances that you measured in **Knowledge and Skill Builders A** and **Knowledge and Skill Builders B** ? Explain your analysis.

What inaccuracies were inherent to measuring distance and time in forming the calculations the values for $\mathbf{R_P}$? Explain.

Since there are inaccuracies in calculating $\mathbf{R_P}$, how accurate a measurement is the equation for predicting times based on desirable distance and rates of speed (velocity)? How can these inaccuracies be minimized? Explain.

Knowledge and Skill Builder E: Predicting Time Based on Power Levels and Distances within the Task

Using the transformed equation for ***time in terms of distance and rates of speed***,

Time $= \dfrac{D}{R_p}$	where $\mathbf{R_p}$ represents velocity for the given power level, $\mathbf{p = \{1, 2, 3, 4, 5\}}$

the programming time needed to make the motor vehicle travel a given distance for a given power level can be calculated.

Examine the terrain task that you need the motor vehicle to travel in competition. Measure off five different distances embedded within your task that you need to traverse. Using the above formula, calculate the time necessary that the motors on the wheels need to be programmed to traverse these distances, for each of the five power levels, $\mathbf{p = \{1, 2, 3, 4, 5\}}$.

Desired Distance	Calculated Time For Motor Power Level = 1	Calculated Time For Motor Power Level = 2	Calculated Time For Motor Power Level = 3	Calculated Time For Motor Power Level = 4	Calculated Time For Motor Power Level = 5
Distance$_1$ = _____					
Distance$_2$ = _____					
Distance$_3$ = _____					
Distance$_4$ = _____					
Distance$_5$ = _____					

Given these five distances that need to be traversed and the time constrains inherent to the task, which power level would be most desirable for your program?

____

____

Knowledge and Skill Builder F: Turning Based on Power Levels and Motor Speeds

In order to make the motor vehicle turn either clockwise or counterclockwise, the most accurate and most efficient method is to make the vehicle change directions about a central point without moving forward, backwards, or to the side. This is done by making the motors attached to the wheels ***rotate*** in opposite directions – one forwards and one backwards.

This method has the following advantages and disadvantages:

1. The vertical and horizontal ***coordinates*** – placement – are not changed on the terrain map.

2. The turn is smooth and easily calculated since it rotates about a fixed point – located at the center of the axle connecting the wheels.

3. By not moving at the same time that the direction is changed has the disadvantage that time may not be used as ***efficiently*** as possible.

4. Experimentation must be done in order to determine the exact time that the wheels need to rotate in opposite directions in order to effect the exact desired ***rotational degree*** change in the ***entire*** motor vehicle.

In this experiment, in order to determine the exact time, for each power setting, that the tires need to rotate to effect the exact desired ***rotational degree*** turn, the following must be done:

1. One wheel's motor must be programmed to rotate ***forward*** and the other motor wheel must be programmed to rotate in ***reverse*** with respect to the exact same power level.

2. The times should be set in small intervals, such as in consecutive increasing intervals of 0.25 of a second.

3. The angle that the motor vehicle turns must be measured, using a protractor, in ***degrees***, with reference to its forward direction, for each of the different time trials, for each of the five power levels, **p = {1, 2, 3, 4, 5}**.

4. It is important to note that trial tests only need to be conducted for turns in either the clockwise or counterclockwise direction inasmuch as turns in the opposite direction are merely a matter of switching (reversing) the wheel rotation direction.

This experiment is recorded on the following page.

How does rounding off errors affect the accuracy of your measurements? Explain.

Right Wheel Forward Left Wheel in Reverse	Power Level = 1	Power Level = 2	Power Level = 3	Power Level = 4	Power Level = 5
Rotation After 0.25 Seconds					
Rotation After 0.50 Seconds					
Rotation After 0.75 Seconds					
Rotation After 1.00 Seconds					
Rotation After 1.25 Seconds					
Rotation After 1.50 Seconds					
Rotation After 1.75 Seconds					
Rotation After 2.00 Seconds					
Rotation After 2.25 Seconds					
Rotation After 2.50 Seconds					
Rotation After 2.75 Seconds					
Rotation After 3.00 Seconds					
Rotation After 3.25 Seconds					
Rotation After 3.50 Seconds					
Rotation After 3.75 Seconds					
Rotation After 4.00 Seconds					
Rotation After 4.25 Seconds					
Rotation After 4.50 Seconds					
Rotation After 4.75 Seconds					
Rotation After 5.00 Seconds					

Knowledge and Skill Builder G: The Equation of Rotation Angle versus Time Based on Power Levels

For each of the motor power level settings, is there a mathematical relationship between the angle that the motor vehicle turns (***rotates***), measured in degrees, and the time that the left and right wheel motors are programmed to run in seconds. Since the turn is smooth, and has a constant arc ratio since the power setting has one constant setting for the entirety of the turn, this equation should be a linear equation.

Using the data collected in **Knowledge and Skill Builder F**, determine the constant ratio of

$$\frac{\textbf{Rotation Angle}}{\textbf{Time}}$$

for each of each of the five power level setting, **p = {1, 2, 3, 4, 5}**.

Left & Right Wheel Motor Power Levels	Equation of Rotation Angle Versus Time
Power Level= 1	
Power Level= 2	
Power Level= 3	
Power Level= 4	
Power Level= 5	

Why might there be inaccuracies resulting from prediction from these equations? Explain.

____

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How do friction and inertia affect predictions from these equations? Explain your analysis.

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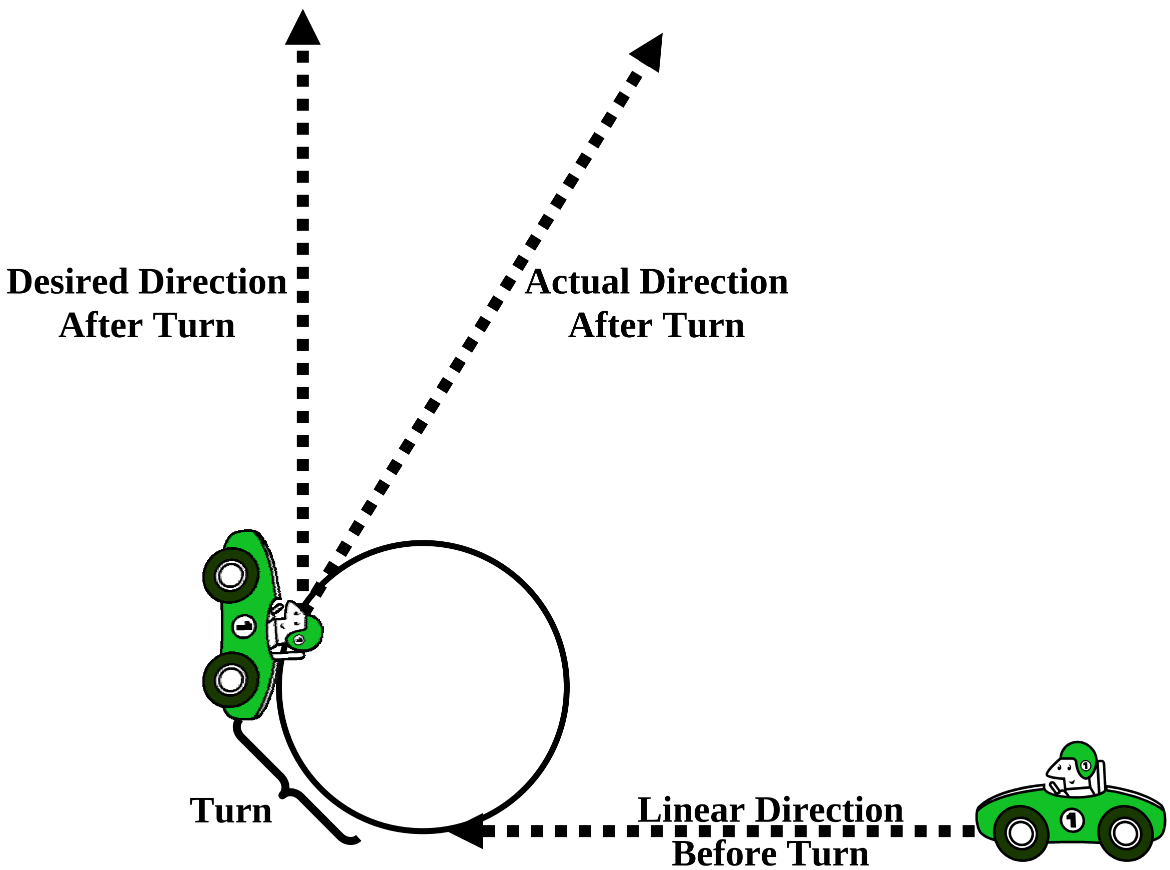
How severely will rounding off errors affect the accuracy of your angular measurements and predictions that could be made using these equations?

After the turn is made, imagine that the motor vehicle is programmed to travel in a straight line (a linear path). How would slight inaccuracies in the angular calculations, over a small distance, affect the long term placement of the car after the turn? Explain your analysis.

<u>Hint</u>: Consider the diagram below and the ***<u>direction</u>*** of ***velocity*** and ***force***.



Knowledge and Skill Builder H: Calculating Turns Based on Power Levels

For each of the motor power level settings, **Knowledge and Skill Builder G** helped determine the mathematical relationship and equation between the angle that the motor vehicle turns (***rotates***), measured in degrees, and the time that the left and right wheel motors are programmed to run in seconds for each of the power level setting, **p = {1, 2, 3, 4, 5}**.

Using these equations of ***angular rotation versus time***, for each of the five power levels **p = {1, 2, 3, 4, 5}**, calculate the time needed for the motor vehicle to rotate each of the following angles: a **45º** turn; a **90º** turn; a **135º** turn; and, a **180º** turn.

Desired Angular Rotation Of Turn	Calculated Time For Motor Power Level = 1	Calculated Time For Motor Power Level = 2	Calculated Time For Motor Power Level = 3	Calculated Time For Motor Power Level = 4	Calculated Time For Motor Power Level = 5
Turn = 45º					
Turn = 90º					
Turn = 135º					
Turn = 180º					

Given the angles the motor vehicle must turn in order to complete any given task (in competition), and given both the time constrains inherent to the task and the extreme importance that must be placed on slight inaccuracies in angles of the turns – since they greatly affect the ***direction*** of velocity and the path of future travel – which power level would be most desirable for your program?

____

____

How might the equations of rotation angle versus time and/or results above be used to determine the time need to turn other angular measurements, such as: a turn or 30º; a turn of 60º; a turn of 120º; a turn of 150º; etc.?

____

____

Knowledge and Skill Builder I: Turns Based on the Wheels Moving Forward at Different Power Levels

The first, most accurate and most efficient, method for making a motor vehicle turn is to make the ***entire*** motor vehicle rotate about the central point located in the middle of the axle connecting the two motorized wheels. This is done by making the motors attached to the wheels ***rotate*** in opposite directions – one forwards and one backwards.

A second method towards making the motor vehicle turn in either a clockwise or a counterclockwise direction, is to operate both wheels' motors in the same direction – both forward or both in reverse – but set them at different power levels.

This method has the following advantages and disadvantages:

1. The motor vehicle is in constant forward motion – although circular in trajectory – so time may be more ***efficiently*** to.

2. By being in constant motion, and moving along the arc of a circle, slight inaccuracies in calculations will have tremendous hazardous long run ramifications on the vertical and/or horizontal components (placement) of location on the terrain map.

3. Experimentation must be done in order to determine the exact time that the wheels need to rotate in opposite directions in order to effect the exact desired ***rotational degree*** change ***and*** the desired movement both horizontally and vertically on the terrain map in the ***entire*** motor vehicle.

In this experiment, in order to determine the exact time, for each power setting, that the tires need to be programmed to rotate according to two different power levels to affect the exact desired ***rotational degree*** turn, the following must be done:

1. Let the five power levels, **p = {1, 2, 3, 4, 5}**, be referred to as $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$, $\mathbf{P_4}$, and $\mathbf{P_5}$ respectively.

2. Set both motor wheels rotating in a forward; but one wheel operating at a higher power level than the other. That is $\mathbf{P_{Left} < P_{Right}}$. For example: the left wheel might be set at a power level of 1; whereas, the right wheel would be set at a power level of 3.

3. Note: For turns to the left, the power level on the right wheel should be set at a higher level than the left wheel; and, vice versa.

4. The angle (in ***degrees***) that the entire motor vehicle turns, as compared to its initial forward direction will be measured for different time trials ***and*** different power levels.

5. The horizontal and vertical ***change*** in location on the terrain map for the ***entire*** motor vehicle will also be measured in centimeters.

Data can be tabulated in a chart such as the following:

Left & Right Power Level Settings for Wheels	Horizontal (x), Vertical (y), and Angular (<) Changes After 1 Second	Horizontal (x), Vertical (y), and Angular (<) Changes After 2 Seconds	Horizontal (x), Vertical (y), and Angular (<) Changes After 3 Seconds	Horizontal (x), Vertical (y), and Angular (<) Changes After 4 Seconds	Horizontal (x), Vertical (y), and Angular (<) Changes After 5 Seconds
P_1 and P_2	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_1 and P_3	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_1 and P_4	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_1 and P_5	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_2 and P_3	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_2 and P_4	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_2 and P_5	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_3 and P_4	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_3 and P_5	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____
P_4 and P_5	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____	Δx = _____ Δy = _____ Δangle = _____

Knowledge and Skill Builder J: Measuring Complete or Half Turns Based on Different Power Levels

For each of combination of the power levels, to calculate turns based on disparate power levels, it facilitates understanding by measuring the diameter of the circular paths of the lower powered (**$diameter_L$**) and higher powered (**$diameter_H$**) wheels traverse, respectively.

This is don't by measuring the distance between the starting point and the ending point of the lower and higher powered wheels, respectively, after have traveled exactly 180 degrees of a complete circular path (a semicircular arc).

These can be tabulated in the following chart:

Right & Left Wheel Power Level Settings	**Measured $Diameter_L$**	**Measured $Diameter_H$**
P_1 and P_2		
P_1 and P_3		
P_1 and P_4		
P_1 and P_5		
P_2 and P_3		
P_2 and P_4		
P_2 and P_5		
P_3 and P_4		
P_3 and P_5		
P_4 and P_5		

Use a centimeter ruler to measure the axle length – the distance between the two wheels on the motor vehicle. Let this quantity be represented by: **L = Axle Length**.

For each combination of disparate power levels, determine the distance difference between these two measurements: **diameter_L** and **diameter_H**. Is this relationship a constant? Explain why or why not this may be the case.

What is the relationship between this distance and the axle length, **L**? Explain.

How do the distances, **diameter_L** and **diameter_H,** relate the circumference of the circular paths that they traverse? What is this ***formula***? Explain this mathematical relationship.

How can the **diameter_L** or **diameter_H** be calculated using this formula and measuring the distance – circumference – of the circular path traversed? Explain your analysis or formula manipulation.

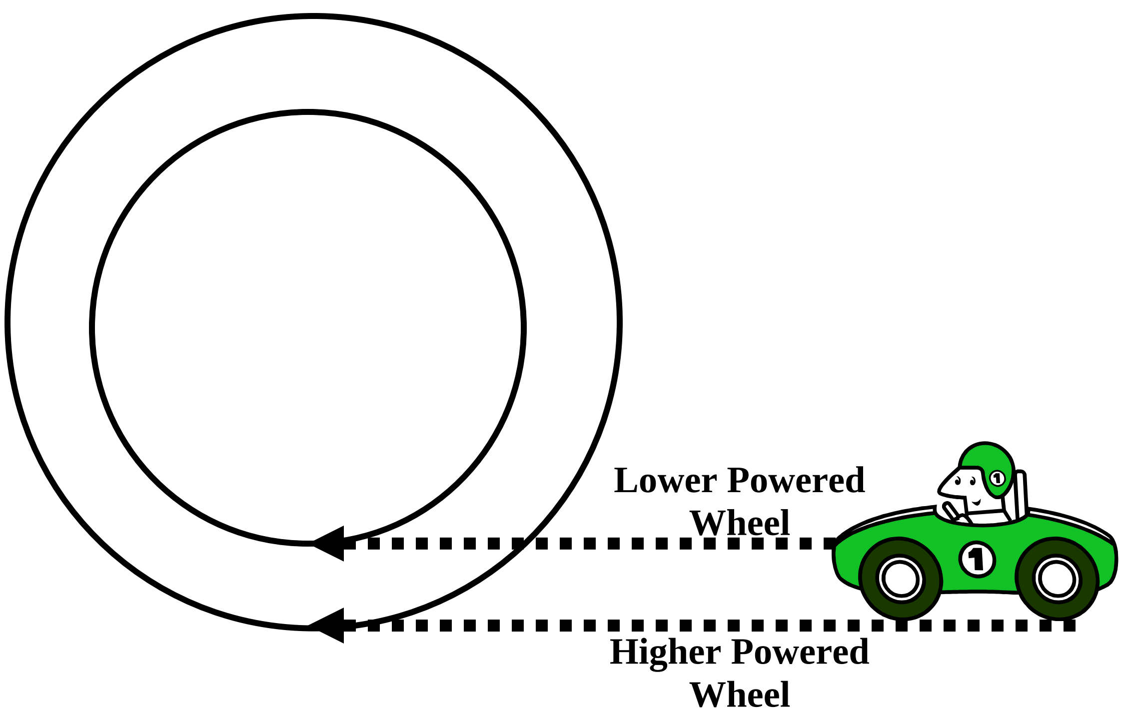
How can the ***rotational sensor*** be used to measure the circular path traversed? Explain how you would program the rotational sensor.

Knowledge and Skill Builder K: Theoretical Mathematics of Turning Based on the Different Power Levels

Now, let use consider the theory of turning based on the wheels being programmed to rotate forward at different power levels:

1. Let the five power levels, **p = {1, 2, 3, 4, 5}**, be referred to as $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$, $\mathbf{P_4}$, and $\mathbf{P_5}$ respectively.
2. Each power level has an associated ***velocity*** – rate of speed ($\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$, $\mathbf{R_4}$, or $\mathbf{R_5}$, respectively), which were derived in **Knowledge and Skill Builder C** – at which the motors on the wheels cause the entire motor vehicle to move forward.
3. Since the power level, and consequently velocity, associated with each individual wheel is programmed to be a constant during any given turn, it can be shown that the vehicle's wheels will move along the arcs of two related ***similar*** concentric circles.
4. The wheel set to the higher power level will move along the arc of the larger circle; whereas, the wheel set to the lower power level will move along the arc of the smaller concentric circle.

The later two properties can be illustrated using the following diagram:



Let $\mathbf{R_L}$ and $\mathbf{R_H}$ represent the rates of speed, as measured in **Knowledge and Skill Builder C**, that the lower powered wheel and the higher powered wheel will make the ***entire*** motor vehicle move respectively.

Since ***Distance equals Rate multiplied by Time***, the distances $\mathbf{D_L}$ and $\mathbf{D_H}$ traveled by the lower powered and higher powered wheels, respectively, are given by:

$$\mathbf{D_L = R_L \bullet time} \qquad \text{and} \qquad \mathbf{D_H = R_H \bullet time}$$

Furthermore, inasmuch as the vehicle's motorized wheels will travel in approximate concentric circular paths, the distances ***between*** these two circles will always differ by the constant amount, **L**, equal to the **axle length** between the two wheels.

Since the quantity of ***time*** is always the same for both wheels at any point in time, within a given angular turn, we can solve both equations for this common variable (***time***) to yield:

$$\mathbf{time = \frac{D_L}{R_L} = \frac{D_H}{R_H}}$$

Using this equation, the fact that the distance around any complete circular path is given by

$$\mathbf{C = \pi d,} \qquad \textbf{Circumference equals pi times the diameter of the circle}$$

and the fact that

$$\mathbf{diameter_H = diameter_L + 2L,} \qquad \text{(as shown by the diagram on the previous page)}$$

this yields the following compound equation:

$$\mathbf{\frac{\pi \bullet diameter_L}{R_L} = \frac{\pi \bullet diameter_H}{R_H} = \frac{\pi \bullet (diameter_L + 2L)}{R_H}}$$

Cross multiplying the first and third portions of the equations, canceling out the value of $\boldsymbol{\pi}$ on both sides of the equation, and solving for the value of $\mathbf{diameter_L}$, yields:

$$\boxed{\mathbf{diameter_L = \frac{2L \bullet R_H}{R_H - R_L}}}$$

Use the ***empirical evidence*** measured in **Knowledge and Skill Builder I**, and the theoretical mathematical formula directly above for $\mathbf{diameter_L}$**,** to fill in the data within the chart on the following page.

Right & Left Wheel Power Level Settings	**Measured $Diameter_L$**	**Measured $Diameter_H$**	**Calculated Value For $Diameter_L$**	**Calculated Value For $Diameter_H$**
P_1 and P_2				
P_1 and P_3				
P_1 and P_4				
P_1 and P_5				
P_2 and P_3				
P_2 and P_4				
P_2 and P_5				
P_3 and P_4				
P_3 and P_5				
P_4 and P_5				

How accurately do the ***theoretical values*** for the **$diameter_L$** correspond to the ***measured values*** for **$diameter_L$**. Explain why there is such accuracy or inaccuracy.

What determination can be made about the ***accuracy*** of turning measurements based on disparate power levels? Explain your analysis.

Linear Movement Based on the Rotation Sensor

To better understand the design challenge, you will first need to gather information in order to build a knowledge base.

Knowledge and Skill Builder L: Investigating Distance Based on the Rotation Sensor

As might be observed using the empirical discoveries of the previous **Knowledge and Skill Builders**, determining linear forward motion, and/or turning ,based on power levels and programming run times will usually result in tremendous inaccuracies in fulfilling task requirements. Consequently, positioning and turning based on the exact rotations of the wheels is desirable.

Programming using the rotation sensor, as opposed to power levels and run times has the following advantages:

1. Friction between the gears becomes a non-factor in calculating run times.
2. Diminishing battery power levels no longer affects the programming tasks since motor speed is no longer the ***independent variable***.
3. Surface friction is no longer a factor inasmuch as programming estimated motor run times are no longer the necessary ***dependent variable***.

A ***rotation sensor*** behaves like the ***odometer*** on a car in that:

1. A rotation sensor measures the number of rotations that the axel, and consequently the wheel, turn.
2. The rotation sensor reads exactly 16 ***positions***, or "***clicks***", per rotation of the wheels' axle.
3. Each click is exactly **one-sixteenth of a rotation**, or a **22.5º degree rotation**, of the wheel.

Experimentation can be used to verify the theoretically calculated number of "clicks", or fractions of a rotation, that must be programmed in order to make the motor vehicle travel a certain distance inherent to the task.

To accomplish this verification, the distance that the motor vehicle travels should be recorded for each of the following different sets of wheel rotations: 5 rotations; 10 rotations; 15 rotations; 20 rotations; 25 rotations; 30 rotations; 35 rotations; 40 rotations; 45 rotations; 50 rotations.

All other distances can be based upon this data.

Record this experimental data in the following table:

Number of Rotations	**Distance Traveled (In Centimeters)**
5 Rotations	
10 Rotations	
15 Rotations	
20 Rotations	
25 Rotations	
30 Rotations	
35 Rotations	
40 Rotations	
45 Rotations	
50 Rotations	

For each of the measured distances able and its corresponding number of rotations, what are the values approximate values of the ratio:

$$\frac{\textbf{Distance}}{\textbf{\# Rotations} \bullet \textbf{Diameter}}$$

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Explain why this might be a constant. Why might this be true?

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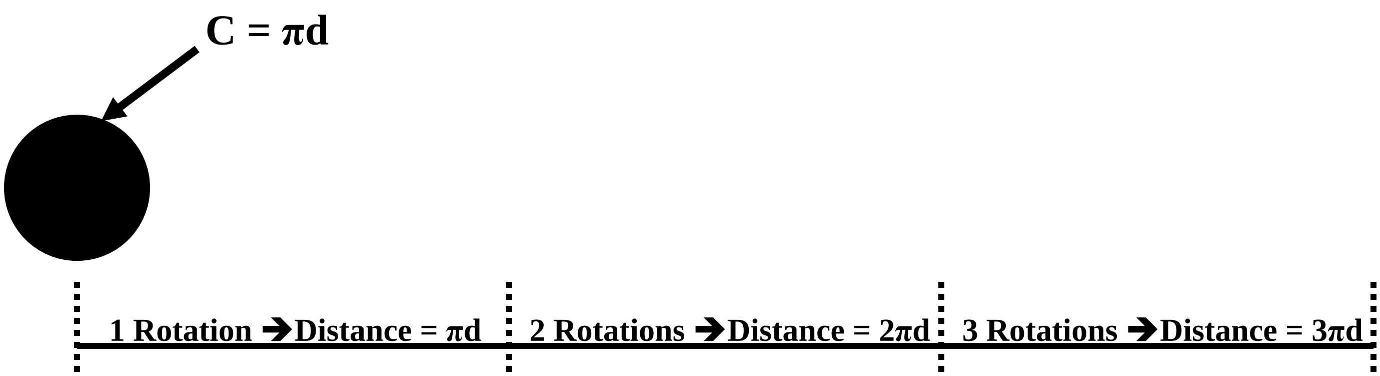
Knowledge and Skill Builder M: Equation of Distance Based on the Rotation Sensor

Did you notice that for each of the measured distances, this ratio is approximately **pi (π)**?

To understand this and derive the equation for linear distance calculation based on the number of rotation of the wheels, let **d** represent the ***diameter*** of the wheel (in centimeters).

For every rotation of the wheel, the distance traveled is merely equal to the circumference of the tire inasmuch as every part of the outside of the tire must touch the surface during one complete rotation. This is illustrated by the diagram below:



Hence, the distance traveled for one rotation of the tires is **πd**.

It follows that for any given number of rotations of the wheel that the distance traveled must equal the product of the number of rotations and the circumference, **πd**.

Given that there are **16 clicks per rotation**, the following formulas evolve:

$$\textbf{Distance} = (\text{\# of Rotations}) \bullet (\pi d) = \frac{\text{\# of Clicks}}{16} \bullet \pi d$$

Solving for the number of rotations and/or clicks that must be programmed to travel any given distance, the following two key equivalent equations evolve that can be used for programming:

$$\boxed{\text{\# of Rotations} = \frac{\text{Distance}}{\pi d}} \quad \text{and} \quad \boxed{\text{\# of Clicks} = \frac{16 \bullet \text{Distance}}{\pi d}}$$

How accurate are your predictions made from these two equations? How do they compare to the distance calculations based on motor power and time?

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Knowledge and Skill Builder N: Investigating Turns Based on the Rotation Sensor

Turning using the rotation sensor, instead of timing and power commands, can be calculated extremely accurately. However, before a formula is derived to calculate the exact number of wheel rotations and/or clicks that are necessary to make a turn of any given angle, it is appropriate to get a "***feel***" through experimentation of making turns using the motor sensor. One slight problem, though extremely minor in comparison to the inherent inaccuracies to using timing commands, is that the rotation sensor is geared to one wheel only. Consequently, one tire must be locked and immobile, so that it doesn't turn, while the other wheel is turning. For the purposes of expedience, trial tests only need to be conducted for turns in either a clockwise or counterclockwise direction inasmuch as turns in the opposite direction are merely a matter of reversing the wheel rotation direction.

In this experiment, the angle that the motor vehicle turns must be measured in ***degrees*** (using a protractor) with reference to its initial forward direction and for different sets of wheel rotations (number of clicks). It must be remembered that ***16 clicks equals 1 complete rotation***.

This experimental data can be recorded in the following table:

Number of Rotations	**Distance Traveled (In Centimeters)**
5 Rotations	
10 Rotations	
15 Rotations	
20 Rotations	
25 Rotations	
30 Rotations	
35 Rotations	
40 Rotations	
45 Rotations	
50 Rotations	

Is there a relationship between the number of "***clicks***" that the wheel rotates and the angle that the entire motor vehicle turns? If so, explain this relationship. Is it consistent?

Knowledge and Skill Builder O: The Equation of Turning Based on the Rotation Sensor

Two measurements are necessary to determine the exact angle that a motor vehicle will turn, when utilizing the rotation sensor:

1. The **axle length** – which is the distance between the two wheels on the motor vehicle. Let the **axle length** be represented by the variable **L**.

2. The **diameter** of the wheel. Let the diameter of the wheel be represented by **d**.

It was shown, in **Knowledge and Skill Builder N**, that distance a wheel travels is given by:

$$\textbf{Distance} = (\#\text{ of Rotations}) \bullet (\pi d) = \frac{\#\text{ of Clicks}}{16} \bullet \pi d$$

Since one tire is be locked and immobile, the other wheel will be turning the ***entire*** motor vehicle along a circle with a ***radius*** equal to the ***axle length***.

Using the fact that the circumference of a circle is given by **C = 2πr** (where **r** is the radius of the circle), it follows that **C = 2π • L**.

Furthermore, since one complete circular path consists of a **360º turn**, the distance that the motor vehicle travels for a turn of any given **angle** can be calculated as follows:

$$\textbf{Distance} = \frac{\text{Angle} \bullet C}{360^\circ} = \frac{\text{Angle} \bullet (2\pi r)}{360^\circ} = \frac{\text{Angle} \bullet (2\pi) \bullet L}{360^\circ}$$

where **L = Axle Length**

Now, the two formulas for **Distance**, it follows that the number of "**clicks**" that must be programmed for the rotation sensor's wheel to turn (in order to make the ***entire*** vehicle turn any given angle) can be determined using to the following formula:

$$\#\text{ of Clicks} = \frac{\text{Angle}}{360^\circ} \bullet 2\pi L \bullet \frac{16}{\pi d} = \frac{\text{Angle}}{360^\circ} \bullet \frac{32L}{d}$$

Consequently, the number of "***clicks***" that rotation sensor's wheel that must be programmed to rotate, to execute any desired angular turn, can be calculated using the following key formula:

***** $$\boxed{\#\text{ of Clicks} = \frac{\text{Angle}}{360^\circ} \bullet \frac{32L}{d}}$$ *****

Knowledge and Skill Builder P: Applying the Equation for Turning Based on the Rotation Sensor

The formula

$$\textbf{\# of Clicks} = \frac{\textbf{Angle}}{\textbf{360º}} \bullet \frac{\textbf{32L}}{\textbf{d}}$$

can now be used to derive the table below which shows the exact the number of "***clicks***" that rotation sensor's wheel that must be programmed to rotate, to make the ***entire*** motor vehicle execute each of the following respective turns: a 30º turn; a 45º turn; a 60º turn; a 90º turn; a 120º turn; a 135º turn; a 150º turn; and, a 180º turn.

Angle of Turn	**30º**	**45º**	**60º**	**90º**	**120º**	**135º**	**150º**	**180º**
# of Clicks to be Programmed	$\frac{8L}{3d}$	$\frac{4L}{d}$	$\frac{16L}{3d}$	$\frac{8L}{d}$	$\frac{32L}{3d}$	$\frac{12L}{d}$	$\frac{40L}{3d}$	$\frac{16L}{d}$

*Note: **L** represents **axle length** and **d** represents the **tire diameter**.*

Program the motor vehicle in order to test the validity of these angle calculations through using experimentation. How accurate are your predictions? Explain.

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How would using **larger or smaller tires** and/or **lengthening or shortening the axle length** affect the number of "clicks" that need to be programmed? Explain your analysis by making reference to the above table or formulas.

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Would having a larger or smaller number of "clicks" needed for programming increase or decrease the accuracy of the turn? Explain.

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Utilizing The Robotic Arm

To better understand the design challenge, you will first need to gather information in order to build a knowledge base.

Knowledge and Skill Builder Q: Investigating Usage of a Robotic Arm Utilizing a Third Motor

In addition to utilizing two separate motors to operate the wheels independently, a third motor can be used to operate a robotic arm attached to the chassis of the motor vehicle.

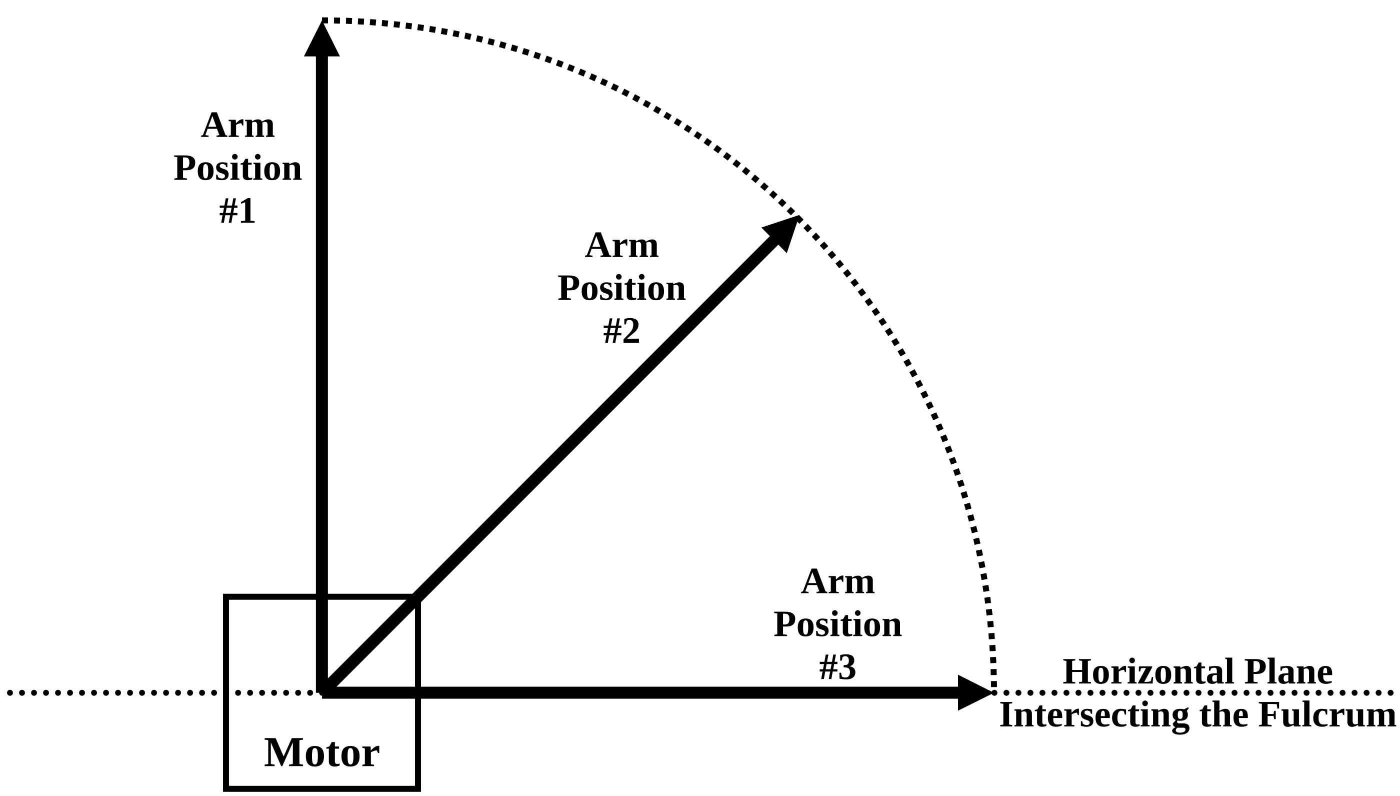
The robotic arm can be programmed in one of two possible different ways:

1. Setting the power level and using either a forward or backward motion to swing the robotic arm forward or backwards in an arc.

2. Determining the number of "clicks", fraction of a rotation, which the robotic arm should swing in the desired direction.

As has been demonstrated, programming utilizing the number of "clicks", or fraction of a rotation, is much more accurately done than programming utilizing power levels and programming run times.

The path that the robotic arm traverses, assuming that the motor vehicle is stationary, can be seen by the following illustration:



Experimentation can be conducted as follows:

1. Set the robotic arm to ***start in a vertical position***.
2. Programming the motor to rotate "**n**" clicks (clockwise or counterclockwise), for each of values of **n = {1, 2, 3, 4, 5, 6, 7, 8}**.
3. Use a centimeter ruler to measure: the constant length (**L**) of the robotic arm; the horizontally placement of the end of the robotic arm; and, the vertical placement of the end of the robotic arm (compared to the horizontal plane intersecting the fulcrum).

This experimental data can be recorded in the following table:

Programmed # of Clicks	**Horizontal Length**	**Vertical Height**
n = 0	**x =** ___________	**y =** ___________
n = 1	**x =** ___________	**y =** ___________
n = 2	**x =** ___________	**y =** ___________
n = 3	**x =** ___________	**y =** ___________
n = 4	**x =** ___________	**y =** ___________
n = 5	**x =** ___________	**y =** ___________
n = 6	**x =** ___________	**y =** ___________
n = 7	**x =** ___________	**y =** ___________
n = 8	**x =** ___________	**y =** ___________

It should be noted that the following are equivalent robotic arm movements:

1. Rotating forward 8 “clicks” is equivalent to rotating backwards 8 “clicks”.
2. Rotating forward 9 “clicks” is equivalent to rotating backwards 7 “clicks”.
3. Rotating forward 10 “clicks” is equivalent to rotating backwards 6 “clicks”.
4. Rotating forward 11 “clicks” is equivalent to rotating backwards 5 “clicks”.
5. Rotating forward 12 “clicks” is equivalent to rotating backwards 4 “clicks”.
6. Rotating forward 13 “clicks” is equivalent to rotating backwards 3 “clicks”.
7. Rotating forward 14 “clicks” is equivalent to rotating backwards 2 “clicks”.
8. Rotating forward 15 “clicks” is equivalent to rotating backwards 1 “clicks”.

If the robotic arm length is short enough, as compared to the vertical height that its fulcrum is off the ground, what combinations of two forward rotations would return the robotic arm into its starting position? What combination of two backwards rotations would accomplish the same task? Explain your analysis.

How would having a longer or shorter robotic arm length (**L**) affect the number of “clicks” that need to be programmed? Explain your analysis.

Would having a larger or smaller number of “clicks” needed for programming increase or decrease the accuracy of measuring the placement of end of the robotic arm? Explain.

Knowledge and Skill Builder R: Equations for Using the Robotic Arm Based on the Rotation Sensor

Let's consider the mathematical theory of using the robotic arm:

1. The length (**L**) of the robotic arm is constant.
2. It is attached to a motor that rotates this arm along a circular path whose radius is equal to the length (**L**) of that robotic arm.
3. Consequently, the distance along the circular arc that the end of the robotic arm travels is given by:

$$\textbf{Distance} = \frac{\textbf{\# of Clicks}}{\textbf{16}} \bullet 2\pi L$$

Two formulas can be developed to measure the horizontal and vertical location of the robotic arm, with reference to the horizontal plane intersecting the fulcrum of the arm.

These formulas are derived:

1. Using simple trigonometric functions.
2. The fact that each "***click***" rotates the motor attached to the robotic arm exactly **22.5º degrees**; and, is one-sixteenth of a complete 360º rotation.
3. Assuming that the robotic arm ***starts in a vertical position***.
4. The motor rotates the robotic arm ***freely*** exactly **n**" clicks either clockwise or counterclockwise from its vertical starting position, with **n** belonging to the set **{1, 2, 3, 4, 5, 6, 7, 8}**.

This analysis yields that programming the motor to rotate "**n**" clicks from its vertical starting position will result in the robotic arm extending the following distances horizontally and vertically above the horizontal plane intersecting the fulcrum (where **L** is the length of the robotic arm) according to the following two trigonometric formulas:

$$\textbf{Horizontal Length} = L \bullet \sin(22.5n) \quad \textbf{and} \quad \textbf{Vertical Height} = L \bullet \cos(22.5n)$$

Actual values for the number of programmed "clicks" (**n),** belonging to the set **{1, 2, 3, 4, 5, 6, 7, 8}** "**clicks**", can be substituted into these two equations to yield the table on the next page:

Programmed # of Clicks	Horizontal Length	Vertical Height
n = 0	**x = 0**	**y = L**
n = 1	**x = 0.3827 • L**	**y = 0.9239 • L**
n = 2	**x = 0.7071 • L**	**y = 0.7071 • L**
n = 3	**x = 0.9239 • L**	**y = 0.3827 • L**
n = 4	**x = L**	**y = 0**
n = 5	**x = 0.9239 • L**	**y = – 0.3827 • L**
n = 6	**x = 0.7071 • L**	**y = – 0.7071 • L**
n = 7	**x = 0.3827 • L**	**y = – 0.9239 • L**
n = 8	**x = 0**	**y = – L**

How accurate do your mathematical calculations and prediction compare to their corresponding measures values? Explain any discrepancy.

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What determination can be made about the ***accuracy*** of rotating the robotic arm based on the rotation sensor? Explain your analysis.

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How do you think the accuracy will change if the robotic arm is programmed utilizing power levels and specific run times? Explain.

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The Optic Light Sensor

To better understand the design challenge, you will first need to gather information in order to build a knowledge base.

Knowledge and Skill Builder S: Using the Optical Light Sensor Based on the Illumination

The ***optic light*** sensor is programmed in much the same way that the rotation sensor is programmed.

The light sensor (optic sensor) interprets the amount of light entering the sensor within a reading range of zero (complete black) through 100 (complete white).

Before hard-coding the programming in order to fulfill the task at hand, it is necessary to determine what a **LIGHT** reading on the task terrain is and what a **DARK** reading on the task terrain is.

To determine these readings:

1. A simple program must be loaded and run which records the ***light sensor's*** readings.
2. Place the light sensor over a **WHITE** area and record this reading for **LIGHT**:

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2. Place the light sensor over a **BLACK** area and record this reading for **DARK**:

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3. Calculate the approximate ***average*** of the **LIGHT** and **DARK** readings:

$$\textbf{Average} = \frac{\textbf{LIGHT Reading + DARK Reading}}{\textbf{2}}$$

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4. Enter this approximate ***average*** as the ***threshold*** in the programming "**WAIT UNTIL**" command in order to allow the light sensor to ***differentiate*** between lightness and darkness on the terrain map.

5. When this approximate average is entered into the "**WAIT UNTIL**" command, the light sensor will define all values ***less than*** this average as being "**DARK**" and all values ***greater than*** this average as being "**LIGHT**".

Once this is hard-coded into a subroutine, existing programs can be modified to start within a white (light) area move linearly forward until they reach a darker (black) area before engaging in further tasks; and vice versa.

For example:

1. The motor vehicle starts in a **LIGHT** area; and is programmed to move initially linearly forward.

2. This forward motion continues ***until*** the ***optical light sensor*** records a reading below its programmed entered threshold – ***below*** the approximate average.

3. The "**WAIT FOR DARK**" function then returns a ***Boolean*** value of "**YES**" value when it reaches this darkened area (or black line).

4. The subroutine of forward movement is terminated; and, ensuing tasks such as following the black line or turning then commence.

Alternatively:

1. The motor vehicle can starts in a **DARK** area; and is programmed to move initially linearly forward.

2. This forward motion continues ***until*** the ***optical light sensor*** records a reading above its programmed entered threshold – ***above*** the approximate average.

3. The "**WAIT FOR LIGHT**" function then returns a ***Boolean*** value of "**YES**" value when it reaches this lighter area.

4. The subroutine of forward movement is terminated; and, ensuing tasks such as following the black line or turning then commence.

How do different ambient lighting conditions, or changing light sources, affect the ***optical sensor*** reading for **LIGHT** and **DARK**? Explain.

How might these lighting conditions affect your programming and threshold values? Explain.

How can the **WAIT FOR DARKER** function be used instead of the **WAIT FOR DARK** function? Why might this function be more unpredictable? Explain.

How can the **WAIT FOR LIGHTER** function be used instead of the **WAIT FOR LIGHT** function? Explain.

How can an "***out-of-range***" threshold be used instead? How might this idea of "***out-of-range***" be programmed and/or hard-coded? Explain.

*__Hint__: In **pseudocode**, the **Boolean logic** schemata might be as follows:*

Start Program
    Run Motor A Forward
    Run Motor C Forward
    Start test loop
        If light sensor is less than a lower range level then exit test loop
        Else
            If light sensor is greater than an upper range level then exit test loop
            Else
                Goto "Start test loop"
            Endif
        Endif
    End test loop
End Program

__Note__: that a **Touch sensor**, which is a calibrated "true/false" switch functions in much the same way that an optic light sensor function. Using the same **Boolean logic** schemata and similar conditional statement, a motor vehicle can be made to turn or back up or turn in order to get around an object once it has run into something.