Hullform resistance rises rapidly with speed and it is clear that at high speeds efficient shapes must be chosen to limit the engine power needed in a design.
A very complex system – we use a computer to search for the optimum (best) value (because the actual equation is not known).
Optimization by sequential steps – each time, the direction of the next step is dictated by the direction and rate of change of the value between the step you are at and the previous step

(Method of "steepest descent")
New design has ½ the resistance but with a larger actual displacement in the water.
A more difficult problem....

Now you have 11 'normal' coins, and one which is either heavier or lighter.
Again you have just three trials.
How can you do it? (It will have to work under any set of circumstances after the first weighing, etc.)
First person to get a correct answer, wins a prize!
Mathematics in Lego™

With thanks to Patricia A. Poggio, Technology Department Chair, Longwood Junior High School
Lego League/FIRST
http://www.usfirst.org/jrobotcs/flego.htm

**FIRST LEGO League**
The *FIRST* LEGO League (FLL), considered the "little league" of the *FIRST* Robotics Competition, is the result of a partnership between *FIRST* and the LEGO Group. FLL extends the *FIRST* concept of inspiring and celebrating science and technology to children aged 9 through 14, using real-world context and hands-on experimentation.

With the help of LEGO® MINDSTORMSTM Robotics Invention SystemTM technology, young participants can build a robot and compete in a friendly, *FIRST*-style robotics event specially designed for their age group. Using LEGO bricks and other elements such as sensors, motors, and gears, teams gain hands-on experience in engineering and computer programming principles as they construct and program their unique robot inventions.
the 2005 Challenge
Ocean Odyssey
Science integration

- Project Overview
- Choose an ocean or sea activity or resource and trace its impact on our oceans’ health, biodiversity, and productivity. Learn what the experts and other groups are doing in this area, and identify a challenge they are facing.
- Create an innovative solution to help them improve the use of this resource or activity, while minimizing the negative impact on our oceans for present and future generations.
- Finally, share what you have learned with others.
- All three parts of your Project will be evaluated during the judging session at an Official FLL Tournament. Refer to your FLL Coaches' Handbook and Project Rubric for details.
The Challenge Field is an obstacle course on a mat. The obstacles are called Mission Models, and the mat is called the Field Mat. Some of the models are secured to the mat using 3M Dual Lock fastening material. The mat must be on a smooth flat surface, and it must be surrounded by border walls to contain all the action.
Optional Table
With safety, weight, height, and cost in mind, a simple design is offered here, but as long as your surface is smooth, and your Border Walls are located properly, how you support the field is up to you.

<table>
<thead>
<tr>
<th>Part</th>
<th>Make From</th>
<th>Dimensions</th>
<th>Paint</th>
<th>Quantity (with light)</th>
<th>Quantity (no light)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table surface</td>
<td>luan</td>
<td>96&quot; X 48&quot;</td>
<td>no</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Long Field Border</td>
<td>two-by-four</td>
<td>96&quot;</td>
<td>yes</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Short Field Border</td>
<td>two-by-four</td>
<td>45-1/8&quot;</td>
<td>yes</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Stiffener</td>
<td>one-by-three</td>
<td>48&quot;</td>
<td>no</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Upright</td>
<td>two-by-three</td>
<td>48&quot;</td>
<td>yes</td>
<td>2</td>
<td>N/A</td>
</tr>
<tr>
<td>Cross beam</td>
<td>two-by-three</td>
<td>99&quot;</td>
<td>yes</td>
<td>1</td>
<td>N/A</td>
</tr>
<tr>
<td>Saw horse</td>
<td>kit</td>
<td>H ~ 24&quot;</td>
<td>no</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 2

Figure 3

Dummy Board
Possible Math Infusion

6.N.14 Convert among fractions, decimals, and percents

6.A.6 Evaluate formulas for given input values (circumference, area, and volume)

6.G.1 Calculate the area of triangles and quadrilaterals

6.M.8 Estimate volume, area, and circumference

7.G.5 Identify the right angle, hypotenuse, and legs of a right triangle

7.M.2 Measure length/distance using non-standard units

8.A.2 Write verbal expressions that match given mathematical expressions

8.A.11 Solve equations/proportions to convert to equivalent measurements within metric and customary measurement systems

8.G.4 Determine angle pair relationships when given two parallel lines cut by a transversal
From Missions to Algebraic Expressions and Solving Equations

Deploy the Submarine
40 points (If the Sub is upright)
35 points (If not upright)
The robot must get onto the Research Vessel and put the Submarine overboard from there. The Sub must be touching blue on the mat.

Clean Up a Cargo Shipping Accident
30 Points
Bonus:
5 Points (each Crate at base)
2 Points (each Crate on table)
The robot must get the Shipping Container and its spilled Crates of plastic bags to Base so they don’t harm any wildlife.
The Shipping Container at Base is worth 30 points. As Bonus Objects, all Crates are worth points anywhere on the table. Crates at Base are worth 5 points each, and other Crates on the table are worth 2 points each. When removing a Crate, the referee takes the one farthest west at that time.
Find and Recover Archaeological Artifacts
35/25 Points
The robot must get the Artifacts completely off the outline of the ancient shipwreck for 25 points, or back to Base for 35 points.

Conduct a Transect Mapping
30 points (Any number of Flags up)
15 points (Each complete east/west row)
To show the location of the ancient shipwreck, the robot must flip transect Flags up. Any number of Flags up at all (of any color) is worth 30 points, and each complete east/west row is worth an additional 15 points.

Protect the Pump Station
40 Points
The robot must get the Protective Structure to straddle the Pump Station, with at least one of the Protective Structure’s foot pads completely past the Pump Station’s south wall. This result is worth 40 points.
Service the Pipeline
40 Points
The robot must install the new segment of pipe so that the team can test the line. When the segment is installed, the team must use hand force at the east end to raise the yellow flags. The yellow flags raised are worth 40 points.

Sample One Species From Among Others
35 Points
The robot must tag the grey Fish only. The grey Fish must be out of alignment with its location mark in some way and all green Fish must remain aligned with theirs. This result is worth 35 points.

Release the Dolphin
25 Points
The robot must get the Dolphin back into the ocean. The Dolphin touching blue on the mat is worth 25 points.
• So, we could turn each "mission" option into part of an algebraic expression

• If we know that we can only travel so far in the allotted amount of time, and that different mission ‘goals’ are so far apart on the playing field, we could see if we could use an algebraic expression to try to **optimize** our possible score!
"Optimization" is an essential part of engineering design

"A lightning strike on a SAAB 2000 commuter aircraft was simulated with a Giga-cell Finite Difference - Time Domain computational model. The results shown in the figure are the time-evolving surface currents on the metal parts of the aircraft, inside and out." -- The design can be altered to minimize damage – a form of using "virtual reality" simulation for optimization!

www.kcse.kth.se/researcharea-design.html
Finite element analysis – sounds fancy, but it’s just optimizing a whole bunch of algebraic equations simultaneously!
For example.....

- A = submarine deployed
- B = shipping container at base
- C = crate at base
- D = crate on table

Can create an expression for points earned by:
\[(A \times 40) + (B \times 30) + (C \times 5) + (D \times 2) = \text{points}\]
But that would mean ...

- Just make A, B, C, D as big as possible and get all the points! Right?
- (Called a "more is better" optimization problem – only way to get more points would be to "move the design space" – e.g. add creates to the game)
- But what if A, B, C, D take time, and we only have a limited amount of time to accomplish our goals?
• A takes 30 sec.
• B takes 15 sec,
• C takes 10 sec.
• D takes 5 sec.
• And we only have one minute – then what? – And remember – there are multiple crates for C and D on the board!

\[(A \times 30) + (B \times 15) + (C \times 10) + (D \times 5) = 60 \text{ sec.}\]

Would think you would just go for the big point items – more bang for the buck!

But what if you could sweep a whole bunch of C or D (crates) at once to the base or onto the table?

-- Now it starts to get complicated – and more fun!
By adding the time factor, we have “design constraints”
What accounts for how much time we have?

- Actually, we want to maximize our points in a set period of time.
- The robot moves and turns with a particular velocity.
- The playing filed has certain dimensions, and a very particular layout.

- **We need to plan our movements (instructions to the robot) very carefully!** (DESIGN)
Goals for this attempt:
- Reef in shallow H₂O or base 40
- Submarine on blue 35 40
- Shark moved, not fish 35
- Dolphin on blue 25
- Artifact off wreck/sea base 25 35
- Transect Flags up 30 45 60 75
- Pump covered 40
- Pipeline flags up 40
- Container at base 30

Crates:
- at base qty __ x 5 = __
- elsewhere qty __ x 2 = __
Technology Standards Addressed

- 5.1 Engineering Design: As a team, students must brainstorm to determine what kind of robot must be built and which missions should be completed in what order to achieve optimal score
- 5.3 Computer technology: Computers used to access information, research problem, present solutions, write programs to control robot, collect data from light sensor
- 5.4 technological systems – Use sensors to control a system; observe results of commands and modify input to achieve desired output.
- 5.7 Management of technology (teamwork)
Math Standards Addressed Through Infusion
LOTS of choices, at every grade level....

- 5.N.6 Understand the concept of rate
- 5.A.1 Define and use appropriate terminology when referring to variables, and algebraic expressions
- 5.A.2 Translate simple verbal expressions into algebraic expressions
- 5.A.3 Substitute assigned values into variable expressions and evaluate using order of operations
- 5.M.1 Use a ruler to measure to the nearest \(\frac{1}{4}, \frac{1}{2}\) inch, and 1/8 inch
- 5.M.8 Determine the tool and technique to measure with an appropriate level of precision: lengths and angles
6.N.16  Apply the distributive property of multiplication over addition to real numbers, including one variable.

6.A.1  Translate two-step verbal expressions into algebraic expressions.

6.S.7  Justify predictions made from data.

6.S.8  List possible outcomes for compound events.

6.S.9  Determine the probability of dependent events.
• 7.A.1 Translate two-step verbal expressions into algebraic expressions
• 7.A.4 Solve multi-step equations by combining like terms, using the distributive property, or moving variables to one side of the equation
• 7.A.6 Evaluate formulas for given input values (surface area, rate, and density problems)
• 7.M.1 Calculate distance using a map scale
• 7.S.11 Design and conduct an experiment to test predictions
• 8.A.1 Translate verbal sentences into algebraic inequalities
• 8.A.2 Write verbal expressions that match given mathematical expressions
• 8.A.11 Solve equations/proportions to convert to equivalent measurements within metric and customary measurement systems
• 8.A.13 Solve linear inequalities by combining like terms, using the distributive property, or moving variables to one side of the inequality (include multiplication or division of inequalities by a negative number)
So, we can pick anything, but let’s start with one pair of standards to see how we could fill in the worksheet.

• Technology standard: 5.1 Engineering design
• Math standard(s):
  
  - 5.A.1 Define and use appropriate terminology when referring to variables, and algebraic expressions
  
  - 5.A.2 Translate simple verbal expressions into algebraic expressions
  
  - 5.A.3 Substitute assigned values into variable expressions and evaluate using order of operations

What are we really talking about here?

MATHEMATICAL OPTIMIZATION IN ENGINEERING DESIGN
So you try it....

- Describe an exercise involving the Lego Odyssey missions and algebraic expressions
- Try addressing the issues listed at the bottom of the form
- Even ignoring this particular Lego contest, you can imagine many similar projects involving robotics and math – for example, see 8th grade teaching portfolio video available at www.amherstschools.org/webpages/rzdrojewski
Friday Afternoon Workshops
Hofstra University

Please come join us for Friday afternoon workshops for 1st and 2nd wave math and science teachers from the following districts; Amityville, Freeport, Hempstead, Roosevelt and Uniondale. All Technology teachers from all MSTP districts are invited to attend technology workshops at Hofstra University.

Dates are – Oct. 21, 2005, Nov. 18, 2005, Dec. 9, 2005 and January 20, 2006. Dates to be determined for the three remaining spring workshops. All workshops will start at 3:30 p.m. and end at 6:00 p.m.

Math Workshops will be in Hagedorn Hall, room to be determined. Science Workshops will be held in the Chemistry and Physics Building, room 207. Technology Workshops will be held in Adams Hall, Room 012. (Directions will be sent to you shortly.)

Please complete the form below and give it to Lois Miceli at the end of today's session or fax it (516 463-4430). You may also email me (soelmm@hofstra.edu) or call (516 463-6482).

Name: ____________________________
Indicate your discipline: ______________
Home Phone Number: _______________________
Email Address: ____________________________

I will attend the October 21st workshop at Hofstra University.
I cannot attend the October 21 workshop however I would like to attend the remaining workshops at Hofstra University.
Friday Afternoon Workshops

For Stony Brook University CSSTs

Please join us for Friday afternoon workshops for 1st and 2nd wave math and science teachers from the following districts:
Brentwood, Longwood, Riverhead, William Floyd, and Wyandanch.
Dates are – October 14, November 18, and December 9, 2005
Math and Science Workshops will be held in the Student Activities Center (SAC), Rooms 304 and 305 (Directions will be sent to you.)
Stony Brook University Workshops will start at 4:30 p.m. and end at 7:00 p.m.
(A light dinner will be provided.)

October 14: MATH-BASED SCIENCE PROBLEMS AND INVESTIGATIONS
This MSTP workshop session will focus on the analysis and solution of Science Problems that require mathematical analysis. Middle school students when engaged in Fermi questions will learn how to determine “orders of magnitude” [Powers of Ten] estimations for interesting questions such as: How many golf balls can fit into the science classroom?

Two other math-based science investigations will be explored. A comparison of 19th century with 21st century methods of determining longitude and latitude will engage students in learning the role of mathematics in the science of navigation. Finally, workshop participants will learn how to use measurements and mathematical models for predicating the trajectories of projectiles.

November 18: DENSITY
This MSTP workshop session will focus on the mathematical relationships between the mass and volume of various materials. Participants will explore two questions: Is the mass/volume relationship the same for all types of materials? What happens to this relationship when an object is cut in half?

Participants will explore how basic algebra is used to understand the physical nature of matter and how to determine % of error.

December 9: MATH-BASED ENGINEERING DESIGN (description forthcoming)

*Friday afternoon workshops for technology teachers will be held at Hofstra University.
Dates are: Oct. 21, Nov. 18, Dec. 9, 2005 and Jan. 20, 2006, from 3:30 – 6:00 p.m.*
Contact Lois Miceli: Fax: 516-463-4430; Email: soelmm@hofstra.edu; Tel: 516-463-6482

You must register for the October 14th workshop at Stony Brook by Friday, Oct. 7.
Please complete this form and give it to Jacqueline Kampf at the end of today's session, fax it to her (631-632-7809), email her (Jacqueline.Kampf@stonybrook.edu), or call her (631-632-6744).
Name:
Discipline:
Home Phone Number:
Email Address:

____ I will attend the October 14th workshop at Stony Brook University.
____ I cannot attend the October 14th workshop, however, I would like to attend the remaining workshops.
Structure for Friday MSTP Workshops, March – May 2006
Stony Brook University

Friday structure- March
1. Review and plan for classroom use of lesson(s). Discuss process (action research) and assessment tools
2. Learn new MST content
3. Review participant created lessons; design lessons using template for follow up lesson (in grade level disciplinary subgroups or by topic)
4. How computer technology can be used for enhancing learning

Deliverable: 1) Template with lesson outline for ensuing lesson that includes identification of evidence of learning to be collected and compiled; 2) Assessment handout: Multifaceted Assessment System and Assessment Options

Friday structure- April
1. Analyze and discuss student work that has been compiled during that past month to seek evidence of student learning (by subgroup)
2. Learn new MST content
3. Work on template for follow up lesson (in grade level disciplinary subgroups or by topic)
4. How computer technology can be used for enhancing learning

Deliverable: Template with lesson outline for ensuing lesson that includes identification of evidence of learning to be collected and compiled

Friday structure- May
1. Analyze and discuss student work that has been compiled during that past month to seek evidence of student learning (by subgroup)
2. Learn new MST content
3. Work on template for follow up lesson (in grade level disciplinary subgroups or by topic)
4. How computer technology can be used for enhancing learning

Deliverable: Template with lesson outline for ensuing lesson that includes identification of evidence of learning to be collected and compiled
Lessons Learned: From MSTP To Intro. to Engineering Courses

[1] Concept of Informed Design:

- Specification of Design Criteria and Constraints
- Explorations of Alternative Designs
- Relevant Knowledge and Skill Builders [KSBs]
- Evaluation of Prototype Designs

[ii] Infusion of Mathematics into Science and Technology Lessons

Types of Infusion Models:

[A] Math-Based Problem Solving Activities:
   Example: Fermi Questions

[B] Science-Based Math Infusion
[1] Mathematics as a Problem Solving Tool
   Example: Longitude Problem
   Example: Pendulum Experiments

[C] Technology-Based Math Infusion
[1] Mathematical Analysis as a Design Tool

Professor Thomas T. Liao
Department of Technology and Society
College of Engineering and Applied Sciences
What is a Fermi Question?

A Fermi question requires estimation of physical quantities to arrive at an answer.

Throughout his work, Fermi was legendary for being able to figure out things in his head, using information that initially seems too meager for a quantitative result. He used a process of "zeroing in" on problems by saying that the value in question was certainly larger than one number and less than some other amount. He would proceed through a problem in that fashion and, in the end, have a quantified answer within identified limits.

In a Fermi question, the goal is to get an answer to an order of magnitude (typically a power of ten) by making reasonable assumptions about the situation, not necessarily relying upon definite knowledge for an "exact" answer.

- A Fermi question is posed with limited information given.
  - How many water balloons would it take to fill the school gymnasium?
  - How many piano tuners are there in New York City?
  - What is the mass in kilograms of the student body in your school?
- A Fermi question requires that students ask many more questions.
  - How big is a water balloon?
  - What are the approximate dimensions dimensions of the gym?
  - What measurement must be estimated using the dimensions of the gym?
  - ... and the list goes on.
- A Fermi question demands communication.
- A Fermi question utilizes estimation.
- A Fermi question emphasizes process rather than "the" answer.
Math-Based Investigations in Earth and Space Science

Investigation #1: Determining the scale of a model globe

[A] Estimate the scale without measurements: ________________________________
Briefly describe estimation technique: ______________________________________

[B] Determine the scale by comparing the diameter of the Earth [8,000 miles] with the
diameter of the globe [to be measured]: ________________________________
Show sample calculation: ________________________________________________

[C] Compare the scales from parts [A] and [B] with the scale that is printed on the globe
in terms of the accuracy and the units used:
____________________________________________________________________

[D] What math concepts were used in this investigation? _______________________

Investigation #2: Determining Longitude at Sea

[A] Using the globe, explain how an accurate clock set to Greenwich time can be used to
determine the longitude of a ship at sea: ________________________________

[B] What math concepts were used in this investigation? _______________________

Investigation #3: Comparing the distances and travel times in the Solar System

[A] With each person on your team representing a planet, position yourselves to create a
scale of the solar system within the size of the room. What scale did your team pick and
why? ________________________________________________________________

[B] Determine the time it takes from sunlight and other forms of Electromagnetic Waves
to travel from the sun to the Earth? (speed of light is 186,000 miles/sec. or 3 x 10^8 m/s)

[C] When NASA needs to communicate with the two Rover robots on Mars, what affects
the time it takes for the radio waves to travel from Earth to Mars?

Estimate the travel time of the radio waves: ________________________________

[D] What math concepts were used in this investigation? _______________________

382
Science 9: Experimenting with a Pendulum  Names ________________________

[I] Question #1: What happens to the period [time for one swing] of successive swings of a pendulum?
Hypothesis: ________________________
Prediction: ________________________
Graph: ________________________

Data [Period for Each Swing]: #1 _______ #2 _______ #3 _______ #4 _______ #5 _______ #6 _______
Graph: ________________________

[ii] Question #2: How does the length of the pendulum affect its Period?
Hypothesis: ________________________

Data: Length [centimeter] | Time [seconds]
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph: ________________________

[iii] Question #3: How does the Mass of the pendulum bob affect its Period?

Data: Mass [grams] | Time [seconds]
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
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<td></td>
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</table>

Graph: ________________________

[IV] Question #4: How does the Amplitude [Angle] of the pendulum affect its Period?

Data: Amplitude [degrees] | Time [seconds]
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

Graph: ________________________
MEASURING MATTER

PROBLEM: Why does a basketball weigh more when it is filled with air? Why does a balloon take up more space when you blow air into it? Why would a basketball, filled with sand, be more dense than one filled with air?

HYPOTHESIS: Come up with an explanation for each of the problems above.

1.

2.

3.

BACKGROUND: We have learned that anything that has mass and takes up space is called matter. Do all materials take up the same amount of space? Do all materials have the same mass? What makes some materials denser than others?

It’s easy to understand that a kilogram of feathers weighs the same as a kilogram of lead. However, a kilogram of feathers takes up much more space than the lead. The lead is said to be more dense. In other words, a lot of material (mass) is squashed into a small amount of space (volume).

Thus, density is related to both mass and volume.

What is this relationship?
Volume of regular solids can be determined by measuring the length, width and height of the object.

Volume = length $\times$ width $\times$ height

The object at right would have the following volume:

3 inches $\times$ 2 inches $\times$ 2 inches = 12 cubic in.

For the materials supplied, measure the masses, volumes, and using your formula, calculate their densities. Put your answers in the chart below.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>MASS (gr.)</th>
<th>VOLUME (cm$^3$)</th>
<th>DENSITY (gr/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>2.</td>
<td></td>
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<td>3.</td>
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<td>4.</td>
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<td>5.</td>
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<td>6.</td>
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<tr>
<td>7.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using the data obtained for the objects in the lab, construct a properly labeled graph for Object vs. Density.
- Put Density on the vertical axis.
- Put Objects on the horizontal axis evenly spaced. (See below)
- Plot 4 "line" graphs. (Aluminum, Brass, Copper, Steel)
- Make sure each of the four lines are labeled.

Make a second graph, also on graph paper, which would show mass vs. volume for the objects in the lab. See below.
Conclusion Questions:

1. Which materials have the same density?

2. How did you determine this by the graphs?

3. What happens to the ratio of mass vs. volume when an aluminum bar is cut in half?

   What is the density of each half?

4. What happens to density when you change the size or shape of samples of the same material?

5. According to the second graph, which material has the greatest density? ________

   How did the graph indicate this?
MSTP Workshop Description

Maps and Math

Introduction
According to the Merriam-Webster dictionary, a map is defined as a representation, usually on a flat surface, of the whole or part of an area. In this context, maps may be further categorized as a set of points, lines and areas defined by position with reference to a specified coordinate system. Many maps are two-dimensional representations of three-dimensional space. The art and science of map-making is referred to as cartography.

Session
This session will focus on the mathematics topics imbedded in the use of maps to locate places on the surface of the Earth. The mathematics topics addressed include “scale”, “area”, “unit conversions”, “fractions”, and “ratios”.

The primary “Connected Mathematics” unit addressed is “Comparing and Scaling”.

The NCTM Principles and Standards 2000 addressed are as follows:
For Comparing and Scaling:
Content Standard – Number and Operations, Algebra
Process Standards – Problem solving, reasoning and proof, communication, connections, and representation
Scale

Objective: To determine the connection between scale and area through the use of maps.

Definition: Map Scale – The relationship between distances on a map and the corresponding distances on the Earth’s surface expressed as a fraction or a ratio.

Activity: Map construction
1) Draw a map that shows the immediate vicinity of your school with respect to the neighboring streets.
2) Draw a second map showing the location of your school with respect to the neighboring townships.
3) Draw a third map showing the location of your school with respect to all of Long Island.
4) Draw a fourth map showing the location of your school with respect to the east coast of the United States.

Observations: (Answer the following questions.)
1) When drawing your maps were they drawn at the same scale as the real world? Explain.

2) What observations can you make regarding the scale of your maps compared to the actual surface being depicted?

3) What observations can you make regarding the level of detail contained in the first map compared to the fourth map?

Comment: Cartographers often describe maps in a relative sense as being either small scale or large scale. A small scale map shows a large area with a small amount of detail. A large scale map shows a small area with a large amount of detail.

4) How would you classify your first and fourth maps? Explain.
Comment: The map scale is typically represented on a map using one of three methods.
(a) Representative fraction – Represents the map scale by a simple fraction or ratio. (i.e. If “one unit” on the map represents “1,000 of the same units” on the actual surface of the Earth, the representative fraction would be 1/1000 (fraction) or 1:1000 (ratio).
(b) Verbal statement – Describes the map scale in words. (i.e. 1:1000 may be described as “1 cm on the map equals 1000 cm on the Earth’s surface or 10 m on the Earth’s surface” or “1 inch on the map equals 1000 inches on the Earth’s surface or 0.0158 miles on the Earth’s surface.”
(c) Graphic scale – An illustration is used to depict distances on the map in common units of measurement.

5) Specify a formula for map scale. Explain how you arrived at this formula.

6) Assume that we have three maps representing an area of the world at scales of 1:100,000; 1:50,000; and 1:25,000. Answer the following based on these three maps.
(a) Which map has the smallest scale relative to the other two?

(b) If the three map scales are represented as fractions, which fraction is the smallest?

(c) What observations can be made regarding scale of the map and the denominator of the fraction?

(d) For each map specify the map distance first in inches with the corresponding Earth distance in inches, feet, and miles. Then specify the map distance in cm with the corresponding Earth distance in centimeters, meters and kilometers. Use the tables provided.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Map Distance (in)</th>
<th>Earth Distance (in)</th>
<th>Earth Distance (ft)</th>
<th>Earth Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:100,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:50,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:25,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale</th>
<th>Map Distance (cm)</th>
<th>Earth Distance (cm)</th>
<th>Earth Distance (m)</th>
<th>Earth Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:100,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:50,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:25,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(e) For each map calculate the square area represented on Earth. Use the table provided.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Area on Earth (ft^2)</th>
<th>Area on Earth (m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:100,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:50,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:25,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(f) Describe the relationship between the map scale and the size of the area represented on the Earth for the three different map scales.

Comments: It is very common to see the map scale represented using a “graphic scale”. The next activity will show how to obtain the map scale as a “representative fraction” based on the graphic scale.

7) Locate the six maps provided in the packet which show the location of William PACA Middle School. The school’s address is 338 Blanco Drive, Mastic Beach, NY 11951.

(a) Comment on the level of detail regarding locating the school in each of the six maps.

(b) Using the metric scale on a ruler determine how many miles on Earth are equivalent to 1 cm on the map.

(c) Convert the number of miles on Earth to the corresponding number of centimeters on Earth. The necessary unit conversions are as follows: 5280 feet = 1 mile; 1 foot = 12 inches; 2.54 centimeters = 1 inch.

(d) Record the map scale as a ratio. (i.e. Distance on the Map: Distance on the Earth). (Note that for map 1 the ratio was approximated to the nearest thousand.)

(e) Determine the map scale as a ratio for the remaining five maps. Record your answers in the table below.

<table>
<thead>
<tr>
<th>Map Number</th>
<th>Distance on Map (cm)</th>
<th>Distance on Earth (miles)</th>
<th>Distance on Earth (cm)</th>
<th>Map Scale Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.05</td>
<td>8,047</td>
<td>1:8000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(f) Describe the relationship between the map scale and the size of the area represented on the Earth for the six maps.
Other Mathematics Topics Linked to Maps

I) Coordinate Systems
   A) The most commonly used coordinate system is latitude and longitude with the Equator and the Prime Meridian used as reference lines.

II) Projections
    A) Representing the true shape of the Earth’s surface (3D) on a 2D map is not a simple task. Various geographic properties may be distorted by projections (i.e. distance and area).
    B) Mercator Projection creates increasing distortions of size as you move away from the equator
       (a) Greenland appears to be the same size as Africa. However, Africa’s land mass is actually 14 times larger.
       (b) The Mercator projection shows shapes pretty much the way they appear on the globe.
    C) Peters Projection is an equal area map.
       (a) Shapes are distorted
Mercator’s Projection

Peters Projection
Breakup of Pangaea

C. 100 Million Years Ago (Cretaceous Period)
Breakup of Pangaea

D. 50 Million Years Ago (Early Cenozoic)
Breakup of Pangaea
Happy Birthday to You!

Professor Thomas R. Sexton, Ph.D.
College of Business
Stony Brook University

What a Coincidence!

* You are at a party with 24 other people.
* You are chatting with Xavier and you discover that you and he have the same birthday.
* "Wow!" you say. "What a coincidence!"
* "Yeah," says Xavier. "What's the probability of THAT?"

That's a Good Question

* What is the probability of THAT?
* It depends on what you mean by THAT.
* There are (at least) three different possible interpretations of THAT.
* The resulting probabilities are very different from one another.
Definitions of THAT

- Def A: "Two randomly selected people (you and Xavier) share the same birthday."
- Def B: "You and at least one other person at the party (not necessarily Xavier) share the same birthday."
- Def C: "There are at least two people at the party (not necessarily you or Xavier) who share the same birthday."

We Need Some Assumptions

- Assumption: The number of people celebrating their birthday is the same every day of the year.
- Rationale:
  - There is no logical reason to assume differently
  - Greatly simplifies the analysis
  - However, actual data show some variation.

Variation by Day

![Graph showing variation by day](attachment:image.png)
Variation by Month

Another Assumption

- Assumption: We can ignore Leap Year Day (February 29).
- Rationale:
  - There are 1461 days, including February 29, in every four-year period.
  - Assuming equally likely birthdays, only one person in 1461, or 0.00068 of the population, will have February 29 as their birthday.

Definition A

- "Two randomly selected people (you and Xavier) share the same birthday."

\[ P(\text{Match}) = \frac{1}{365} = 0.00274 \]
Definition B

- "You and at least one other person at the party (not necessarily Xavier) share the same birthday."
- Answer depends on \( n \), the number of people at the party.
- Easier to find the probability of no match and subtract from 1.

Probability of No Match

\[
\begin{align*}
\text{For } n = 2: & \quad P(\text{No Match}) = \frac{364}{365} \\
\text{For } n = 3: & \quad P(\text{No Match}) = \left( \frac{364}{365} \right)^2 \frac{364}{365} \\
\text{For } n = 4: & \quad P(\text{No Match}) = \left( \frac{364}{365} \right)^3 \frac{364}{365} \\
\text{In general: } & \quad P(\text{No Match}) = \left( \frac{364}{365} \right)^{n-1} \frac{364}{365}
\end{align*}
\]

Probability of at Least 1 Match

\[
P(\geq 1 \text{ Match}) = 1 - \left( \frac{364}{365} \right)^{n-1}
\]

For \( n = 25 \),
\[
P(\geq 1 \text{ Match}) = 1 - \left( \frac{364}{365} \right)^{24}
\]

\[
P(\geq 1 \text{ Match}) = 0.0637
\]
Definition C

- "There are at least two people at the party (not necessarily you or Xavier) who share the same birthday."
- Answer depends on $n$, the number of people at the party.
- Easier to find the probability of no match and subtract from 1.

Probability of No Match

$n = 2$: $P(\text{No Match}) = \frac{364}{365}$

$n = 3$: $P(\text{No Match}) = \frac{364 \times 363}{365 \times 364} \times \frac{364 \times 363 \times 362}{365 \times 364 \times 363}$

$n = 4$: $P(\text{No Match}) = \frac{364 \times 363 \times 362 \times 361}{365 \times 364 \times 363 \times 362} \times \frac{364 \times 363 \times 362 \times 361 \times 360}{365 \times 364 \times 363 \times 362 \times 361}$

$P(\text{No Match}) = \frac{364 \times 363 \times 362 \times 361 \times 360}{365 \times 364 \times 363 \times 362 \times 361} \times \frac{364 \times 363 \times 362 \times 361 \times 360 \times 359}{365 \times 364 \times 363 \times 362 \times 361 \times 360}$
Probability of at Least 1 Match

\[ P(\geq 1 \text{ Match}) = 1 - \frac{(364)(363)\cdots(365-n+1)}{(365)^n-1} \]

For \( n = 25 \), \( P(\geq 1 \text{ Match}) = 1 - \frac{(364)(363)\cdots(341)}{(365)^{24}} \)

\[ P(\geq 1 \text{ Match}) = 0.5687 \]

Simulation

- We can use Excel to compute the probability of at least one match for any given \( n \).
- Spreadsheet illustrates the simulation for \( n=23 \), the smallest integer such that the probability of at least one match exceeds 0.5.
Lessons Learned

- Careful use of language is very important.
- Sometimes, it's easier to solve a related problem.
- Simulation can be useful for estimating probabilities.
- Some apparently surprising events are not really that surprising.
APPENDIX
Teacher Notebook-Math

Purpose: Use this form to keep notes about MST activities on an ongoing basis and to use as a reminder during discussions with faculty team visitor and at MSTP school meetings.

Name: __________________________ District __________________________ Date ___/___/___

1. Which curriculum did you use?
CMP ( ) IMaST ( ) MiC ( ) Teacher-developed ( ) Other ( ) Describe: ______________________

2. What topics, modules, and major concepts were taught?

   

3. What progress or change has been made toward piloting the curriculum plans in Mathematics?

   

4. Describe anything going on in the school that might be supporting or challenging implementation of team MSTP plans.

   

5. Describe any current recruitment or training activities of Second Wave Teachers.

   

6. In what activities have CSTEP students been involved?

   

7. Please record insights/ideas/challenges you have regarding MSTP progress.

   

MSTP M. notebook 2004-2005
Teacher Notebook - Science

Purpose: Use this form to keep notes about MST activities on an ongoing basis and to use as a reminder during discussions with faculty team visitor and at MSTP school meetings.

Name: __________________________ District __________________________ Date ______/____/____

1. Which curriculum did you use?
   CMP ( )  IMaST ( )  MiC ( )  Teacher-developed ( )  Other ( )  Describe: ________________

   Which units or sections? _______________________

2. What topics, modules, and major mathematics concepts were taught?

   ___________________________________________________________________

3. What progress has been made toward piloting curriculum plans?
   Please describe progress in integrating mathematics into your science class.

   ___________________________________________________________________

4. Describe anything going on in the school that might be supporting or challenging implementation of team MSTP plans.

   ___________________________________________________________________

5. Describe any current recruitment or training activities of Second Wave Teachers.

   ___________________________________________________________________

6. In what activities have CSTEP students been involved?

   ___________________________________________________________________

7. Please record insights/ideas/challenges you have regarding MSTP progress.

   ___________________________________________________________________

MSTP S. notebook 2004-2005
Teacher Notebook - Technology

Purpose: Use this form to keep notes about MST activities on an ongoing basis and to use as a reminder during discussions with faculty team visitor and at MSTP school meetings.

Name: __________________________ District __________________________ Date _____ / _____ / _____

1. Which curriculum did you use?
CMP ( )  IMaST ( )  MiC ( )  Teacher-developed ( )  Other ( )  Describe: __________________________

Which units or sections? __________________________

2. What topics, modules, and major mathematics concepts were taught?


3. What progress has been made toward piloting curriculum plans?
Please describe progress in integrating mathematics into your technology class.


4. Please describe anything going on in the school that might be supporting or challenging implementation of team MSTP plans.


5. Please describe any recruitment or training activities of Second Wave Teachers.


6. In what activities have CSTEP students been involved?


7. Please record insights/ideas/challenges you have regarding MSTP progress.


MSTP T. notebook 2004-2005
CSST Report - Faculty Note Sheet

Please use this form to help you collect information about the project activities of your District. Findings should be entered on online survey forms when complete.

Name: ___________________ District ___________________ Date _____/_____/_____

MATHEMATICS

1. Which curriculum is being used in Mathematics?
   CMP ( )  IMaST ( )  MiC ( )  Teacher-developed ( )  Other ( ) _______________________

   Modules or units _______________________

2. Since your last report, what progress or changes have been made toward piloting the Math curriculum?

   ________________________________________________________________

3. Does this implementation match curriculum plans as submitted in fall 2004?
   YES ( )  NO ( )  SOMEWHAT ( )

SCIENCE

4. Which curriculum is being used in Science?
   CMP ( )  IMaST ( )  MiC ( )  Teacher-developed ( )  Other ( ) _______________________

   Modules or units _______________________

5. Since your last report, what progress or changes have been made toward piloting the Science curriculum?

   ________________________________________________________________

6. Does this implementation match curriculum plans as submitted in fall 2004?
   YES ( )  NO ( )  SOMEWHAT ( )
TECHNOLOGY

7. Which curriculum was used in Technology?
   CMP ( )  IMaST ( )  MiC ( )  Teacher-developed ( )  Other ( )
   Modules or units__________________________

8. Since your last report, what progress or changes have been made toward piloting the Technology curriculum?

   ________________________________________

9. Does this implementation match curriculum plans as submitted in fall 2004?
   YES ( )  NO ( )  SOMEWHAONLY ( )

10. Please describe anything going on in the school that might be supporting or challenging implementation of team MSTP plans.

   ________________________________________

SECOND WAVE RECRUITMENT

11. Have there been any activities to recruit or train Second Wave Teachers since your last report?
   YES ( )  NO ( )

12. If yes, please describe these activities (e.g., What has been done? How many new teachers have been recruited? How many trainings have there been to date? etc.)

   ________________________________________

13. If not, why haven’t there been any such activities?

   ________________________________________
PARENTAL INVOLVEMENT
14. Have there been any activities to involve parents since your last report?
   YES ( )       NO ( )

15. If yes, please describe these parent activities.

16. If not, why haven’t there been any such activities?

CSTEP INVOLVEMENT
17. What activities have CSTEP students been involved in?

18. In general, what activities have been most successful?

GENERAL ADVISEMENT
19. What additional support would you recommend for the District?

20. Please record insights/ideas/challenges regarding MSTP progress.
Math Goals and Standards:
Goals should be clearly tied to the lesson, be specific, and be linked to the Standards. The specific Learning Standard(s) (e.g., NYSED, NCTM, etc) associated with each goal should be listed. Note the content area and the standard number. Ex. NYSED MST 3, etc.

Describe the Math Lesson: Use as much detail as possible. The description should be comprehensive and understandable by another teacher.

Describe how this lesson differs from how you would have taught the same material prior to MSTP: Explain any changes you have made to your instruction, as it relates to this lesson, as a result of participating in MSTP.

Content:
- What Math content is covered in this lesson? Elaborate/specify which Math topics are covered.
- How has this lesson been enhanced?
- Is Science content included in this lesson?
- Is Technology content included in this lesson?

Pedagogy:
- How is this content delivered in an inquiry-based way? Describe how teaching methods are inquiry-based.

Anticipated Student Outcomes: Clarify what types of Procedural Fluency, Conceptual Understanding, and Problem Solving are expected from students as a result of this lesson.
- Procedural Fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Can students do the calculations to solve the problem?
- Conceptual Understanding: ability to identify and apply principals, know and apply facts and definitions, and compare/contrast concepts. Do students understand the concept?
- Problem Solving: ability to analyze problems to choose useful strategies; to recognize when and where to use concepts and procedures (and when and where not to). Can students apply the math?

Assessment:
METHODS - List the ways you assessed student understanding in the 3 outcome areas.
OUTCOMES – List student outcomes exhibited (findings from assessment) in each area.
**INFORMED MATH INQUIRY**

**Lesson Title:**

**Teacher:**

**School:**

**Appropriate Grade Level(s):** 5 6 7 8

**Length of time to complete lesson (check one):** __1 period__ __2 periods__ __other (specify:__)__

**Math Goal(s) for the Lesson:**

________________________________________________________________________

**List the Math Standard(s):**

________________________________________________________________________

**Curriculum used/adapted for the Lesson:**

- [ ] MiC
- [ ] CMP
- [ ] IMAST
- [ ] Teacher Developed
- [ ] Other (please describe):

**Describe the Math Lesson:**

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

**Describe how this lesson differs from how you would have taught the same material prior to MSTP:**

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

---

<table>
<thead>
<tr>
<th>Content:</th>
<th>Pedagogy:</th>
<th>Anticipated Student Outcomes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>What Math content is covered in this lesson?</td>
<td>How is this content delivered in an inquiry-based way?</td>
<td>What <strong>procedural fluency</strong> (computational skill) is expected?</td>
</tr>
<tr>
<td>How has this lesson been enhanced?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Is Science content included in this lesson?  
  ____Yes  ____No | | What **conceptual understanding** (comprehension of the concept) is expected? |
| Is Technology content included in this lesson?  
  ____Yes  ____No | | What **problem solving** (application) is expected? |
Math Reporting Activities

Background Information: Describe any background knowledge that students needed to complete this lesson.

Preconceptions: What preconceptions (or hypotheses) did students have regarding this concept in Math? in Science/Technology?

Materials: What materials were used to complete this lesson?

Assessment:

METHODS: How did you assess each of the following?

1. Procedural fluency: ____________________________

2. Conceptual understanding: ____________________

3. Problem solving: _____________________________

OUTCOMES: What did student outcomes actually look like?

1. Procedural fluency: ____________________________

2. Conceptual understanding: ____________________

3. Problem solving: _____________________________

Generalization: How did you assist students in generalizing Math, Science/Technology conclusions beyond this classroom activity?

References and support: List any references used to develop or support this lesson.

Reflections: How did the implementation differ from your expectations? How would you change your plans to improve the lesson?
“INFORMED SCIENCE/TECHNOLOGY INQUIRY”

Lesson Title: ___________________________ Teacher: ___________________________ School: ___________________________

Appropriate Grade Level(s): 5 6 7 8

Length of time to complete lesson (check one): _1 period _2 periods _other (specify: ____________)

Primary Academic Area (circle one): Mathematics Science Technology Other: ___________________________

Science/Tech Goal(s) for the Lesson:

______________________________

______________________________

List the Science/Tech Standard(s):

______________________________

Math Goal(s) for the Lesson:

______________________________

List the Math Standard(s):

______________________________

Describe the Math-Infused Science/Tech Lesson:

________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________________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Math/Science/Tech Goals and Standards:
Goals should be clearly tied to the lesson, be specific, and be linked to the Standards. The specific Learning Standard (e.g., NYSED, etc) associated with each learning goal should be listed. Note the content area and the standard number. Ex. MST 4.3, etc.

Describe the Math-Infused Science/Tech lesson: Use as much detail as possible. The description should be comprehensive and understandable by another teacher.

Content:
- What Math content is covered in this lesson? Elaborate/specify which Math topics are covered.
- Is the Science/Technology content dependent upon (cannot be understood without knowing the math) or enhanced by (math helps but is not essential for doing the Science/Tech) the infusion of this Math?
- Explain your response above. How does the Math content inform Science/Tech knowledge?

Pedagogy:
- How is this content delivered in an inquiry-based way? Describe how teaching methods are inquiry-based.
- Is the math taught as a stand-alone component, or is it taught within the context of Science/Tech?

Anticipated Student Outcomes: Clarify what types of Procedural Fluency, Conceptual Understanding, and Problem Solving are expected from students as a result of this lesson.
- Procedural Fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Can students do the calculations to solve the problem?
- Conceptual Understanding: ability to identify and apply principals, know and apply facts and definitions, and compare/contrast concepts. Do students understand the concept?
- Problem Solving: ability to analyze problems to choose useful strategies; to recognize when and where to use concepts and procedures (and when and where not to). Can students apply the math?

Assessment:
METHODS - List the ways you assessed student understanding in the 3 outcome areas.
OUTCOMES – List student outcomes exhibited (findings from assessment) in each area.
Math Infusion Reporting Activities

Background Information: Describe any background knowledge that students needed to complete this lesson.

Preconceptions: What preconceptions (or hypotheses) did students have regarding this concept in Math? in Science/Technology?

Materials: What materials were used to complete this lesson?

Assessment:

METHODS: How did you assess each of the following?
1. Procedural fluency: 
2. Conceptual understanding: 
3. Problem solving: 

OUTCOMES: What did student outcomes actually look like?
1. Procedural fluency: 
2. Conceptual understanding: 
3. Problem solving: 

Generalization: How did you assist students in generalizing Math, Science/Technology conclusions beyond this classroom activity?

References and support: List any references used to develop or support this lesson.

Reflections: How did the implementation differ from your expectations? How would you change your plans to improve the lesson?
“INFORMED SCIENCE/TECHNOLOGY INQUIRY”

Lesson Title: ........................................ Teacher: ........................................ School: ........................................
Appropriate Grade Level(s): 5 6 7 8 Length of time to complete lesson (check one): __1 period __2 periods __other (specify: ____________)
Primary Academic Area (circle one): Mathematics Science Technology Other: ........................................

Science/Tech Goal(s) for the Lesson:

List the Science/Tech Standard(s): ........................................

Math Goal(s) for the Lesson:

List the Math Standard(s): ........................................

Describe the Math-Infused Science/Tech Lesson:


Content:

What Math content is covered in this lesson?


Is the Science/Technology content dependent upon or enhanced by the infusion of this Math?

__enhanced __dependent

Explain your response above. How does the Math content inform Science/Tech knowledge?


Pedagogy:

How is this content delivered in an inquiry-based way?


Is the math taught as a stand-alone component or is it taught within the context of Science/Tech?

__stand-alone __within Sci/Tech context

Anticipated Student Outcomes:

What procedural fluency (computational skill) is expected?


What conceptual understanding (comprehension of the concept) is expected?


What problem solving (application) is expected?
Math Infusion Reporting Activities

**Background Information:** Describe any background knowledge that students needed to complete this lesson.

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**Preconceptions:** What preconceptions (or hypotheses) did students have regarding this concept in Math? in Science/Technology?

---

**Materials:** What materials were used to complete this lesson?

---

**Assessment:**

**METHODS:** How did you assess each of the following?

1. Procedural fluency: ________________________________
2. Conceptual understanding: _________________________
3. Problem solving: ________________________________

**OUTCOMES:** What did student outcomes actually look like?

1. Procedural fluency: ______________________________
2. Conceptual understanding: _______________________
3. Problem solving: _______________________________

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**Generalization:** How did you assist students in generalizing Math, Science/Technology conclusions beyond this classroom activity?

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**References and support:** List any references used to develop or support this lesson.

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**Reflections:** How did the implementation differ from your expectations? How would you change your plans to improve the lesson?

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