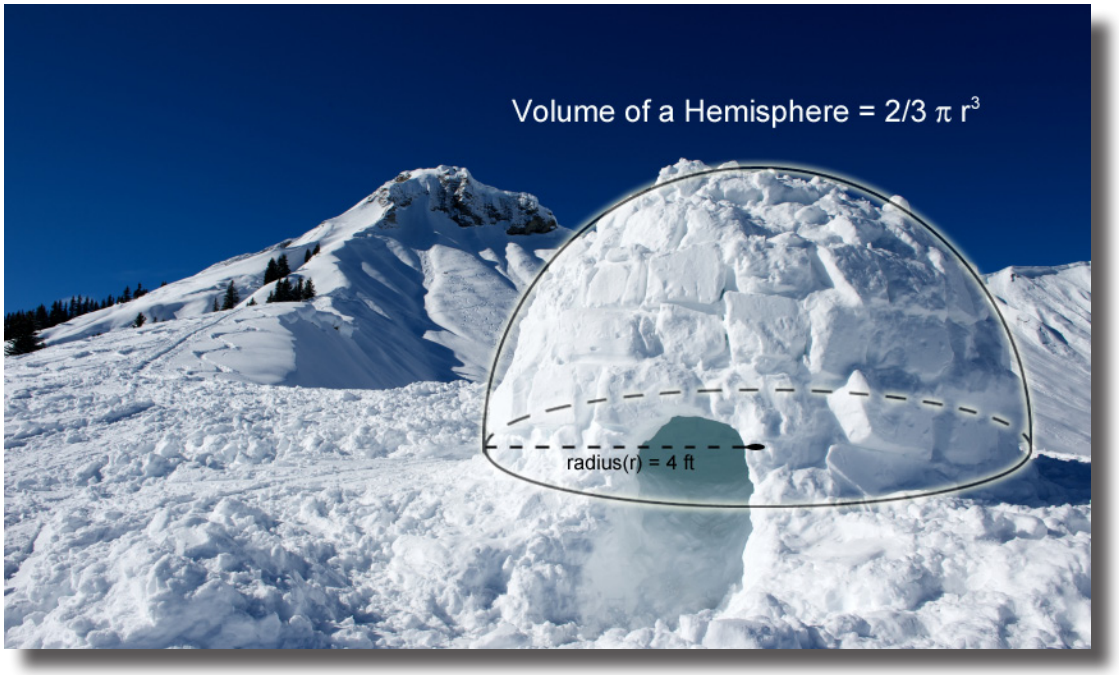
KSB* 1

(*KNOWLEDGE AND SKILL BUILDER)

GEOMETRIC SHAPES



STUDENT NAME: _____

PERIOD: _____

SCHOOL: _____

DATE: _____

Hofstra University Center for Technological Literacy
Simulations and Modeling for Technology Education

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SURFACE AREA AND VOLUME CALCULATIONS

(Please be sure to attach all your drawings and your calculations to this booklet after the last page.)

IN THIS KSB, EACH OF THE FOLLOWING KEY IDEAS WILL BE EXPLAINED CLEARLY; FOR NOW, JUST READ THEM OVER BRIEFLY.

1. Volume is a measure of filling an object and surface area is a measure of wrapping an object.
2. Given the outside dimensions and the mathematical formulas for the volume of each shape, correctly calculate the volume of four geometric shapes: a cube, a sphere, a square-based pyramid, and a cylindrical prism.
3. Given the outside dimensions and the mathematical formulas for surface area for each shape, correctly calculate the surface area of four geometric shapes: a cube, a hemisphere, a square-based pyramid, and a cylindrical prism.

The formulas for surface area and volume of a cube, sphere, cylinder, and square-based pyramid, are:

CUBE: Volume (V) of a cube is $V = s^3$
The surface area (SA) of a cube is $SA = 6*s^2$

CYLINDER: Volume of a cylinder is $V = \pi r^2 h$
The surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$

SPHERE: Volume of a sphere is $V= 4/3\ \pi r^3$
The surface area of a sphere is $SA = 4\ \pi r^2$

HEMISPHERE: Volume of a hemisphere (half a sphere) is $V= 2/3\ \pi r^3$
The surface area of the hemisphere is: $SA_{hemisphere} = 2\ \pi r^2 + \pi r^2 = 3\ \pi r^2$

SQUARE-BASED PYRAMID: Volume of a square-based pyramid is $V = (1/3)\ b^2h$.
The surface area of the square-based pyramid is $SA = 2bs+b^2$

DESIGNING A SURVIVAL SHELTER

To design a survival shelter, you must decide first how much room you need to accommodate the surviving team members. Think about how tall and how wide (from shoulder to shoulder) your team members are. Your teacher may want you to take some shoulder-to-shoulder measurements of your team members, or your teacher may provide you with a set of average measurements that you might use instead.

Figure out how much floor area you will need so that all four can sleep side by side and sit up comfortably.

The minimum floor area of your shelter will need to be at least:

_____ square feet.

POINTS TO PONDER

Next, decide how tall you will make your shelter. You may want to make the shelter tall enough for people to stand upright. That's your choice – but realize that if you make it taller than absolutely necessary, it may have more surface area, may be more difficult to construct, may require more materials, and may require more construction time. However, it will likely be more comfortable. So, how tall would you like to make your shelter?

The shelter will be at least:

_____ feet tall.

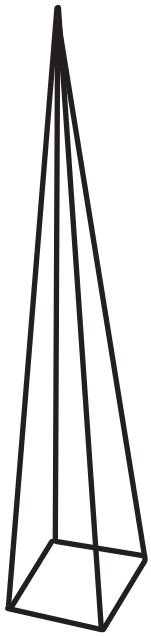
Now decide what **shape** you want to make your shelter. The shape matters. Some shapes have more surface area than other shapes even though they can contain the same volume.

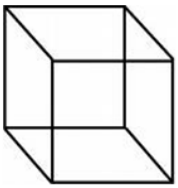
***HERE IS SOME GUIDANCE FOR YOU*: Think about volume (V) as a measure of filling an object and surface area (SA) as a measure of wrapping an object.**

POINTS TO PONDER

Here is an interesting example. These two shapes have exactly the same volume. They both contain 300 cubic feet of space. HOWEVER, this cube has a surface area of 269 ft^2 while this particular pyramid has a surface area of 466 ft^2. Quite a difference.

The SHAPE MATTERS!!





Length of each side = 6.7 ft.

Base side length = 4 ft.
Slant height = 56.29 ft.
Height = 56.25 ft.

In designing your shelter, you want to be able to house your four team members, but at the same time, *minimize the surface area*

WHY DO WE EVEN CARE about minimizing the surface area of the shelter? What do you think the reasons are? Here are some possible reasons. Circle those that you think make sense.

1. The force on the shelter due to wind will be smaller if the surface area is smaller.
2. Shelters with smaller surface areas have a nicer appearance.
3. Heat will flow out of the shelter more slowly if the walls have a smaller surface area.
4. Minimizing the surface area of a shelter will allow the inhabitants to work together more easily.

It's important that you understand the importance of minimizing the surface area of the shelter. Discuss these choices with your team members and your teacher.

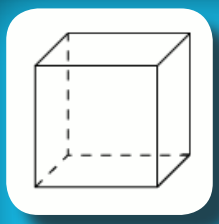
FORMULAS FOR SHELTER DESIGN

In designing your shelter, investigate four different possible shapes: A cube, a cylinder, a sphere (actually a hemisphere), and a square-based pyramid.

Your investigation will involve calculating the volume and the surface area of each of the shapes once you are given their dimensions.

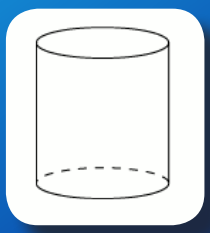
After you do these investigations, you'll decide on the shelter shape.

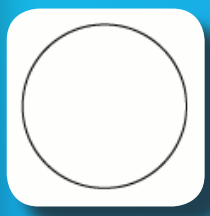
Here are formulas that you may need to use to do your volume and surface area calculations.



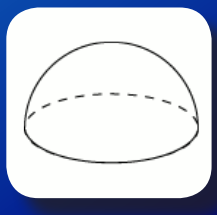
CUBE: Volume (V) of a cube is $V = s^3$
The surface area (SA) of a cube is $SA = 6*s^2$

CYLINDER: Volume of a cylinder is $V = \pi r^2 h$
The surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$

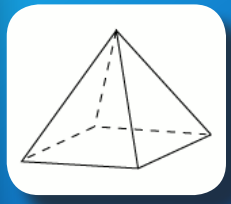




SPHERE: Volume of a sphere is $V = 4/3\ \pi\ r^3$
The surface area of a sphere is $SA = 4\ \pi\ r^2$



HEMISPHERE: Volume of a hemisphere (half a sphere) is $V = 2/3\ \pi\ r^3$
The surface area of the hemisphere is: $SA_{hemisphere} = 2\ \pi\ r^2 + \pi\ r^2 = 3\ \pi\ r^2$



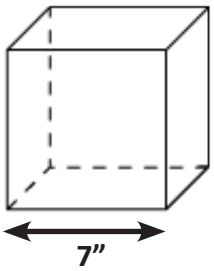
SQUARE-BASED PYRAMID: Volume of a square-based pyramid is $V = (1/3)\ b^2h$.
The surface area of the square-based pyramid is $SA = 2bs+b^2$

MATH AT WORK: *SURFACE AREA AND VOLUME OF A CUBE*

First practice finding the surface area and volume of a cube. Perhaps you may have learned these formulas in your math class. Here's a chance to apply them to a practical design problem (your shelter design).

The formula for the volume (V) of a cube is $V=s^3$, where s stands for the length of any of the sides. The formula for the surface area (SA) of a cube is $SA = 6s^2$.

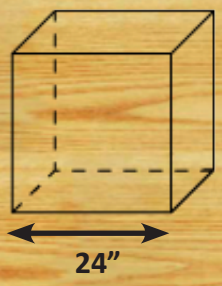
Here is an example: A cube has a side length of 7 inches. To find the volume, use the formula $V=s^3$. Substitute 7 for s, and therefore V= 7 x 7 x 7= 343 cubic inches. To find the surface area of the cube, use the formula $SA = 6s^2$. Substitute 7 for s, and therefore SA = 6x7x7= 294 square inches.



CHECK YOUR UNDERSTANDING

If a cube has a side length of 24”, figure out what the volume and the surface area are.

Write your answers below.
You can use a calculator.

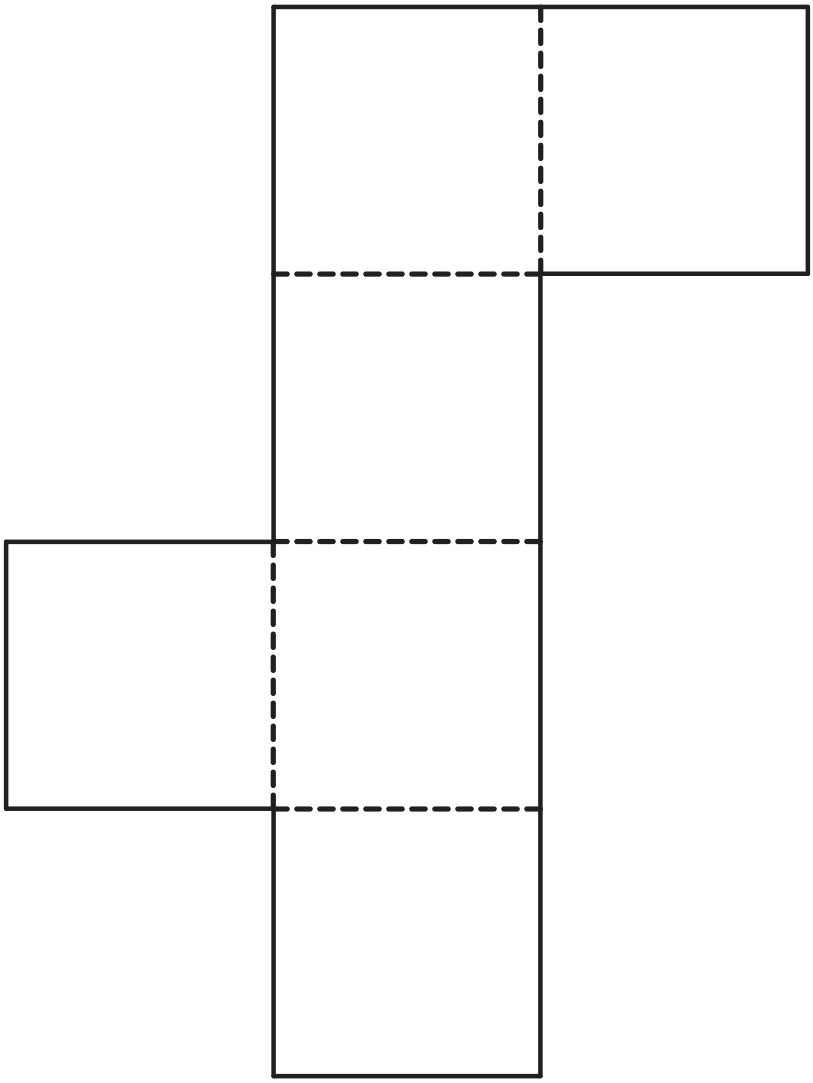


V = _____ **SA =** _____

FOLDING SHAPE NETS: *NET OF A CUBE*

This diagram below shows you what a cube looks like when it is cut and stretched out. In math, this kind of drawing is referred to as a "**net.**" In technology classes, it is often called a "stretch out" or a "development drawing."

Can you visualize that when the figure is folded along the dotted lines, it reforms the cube?

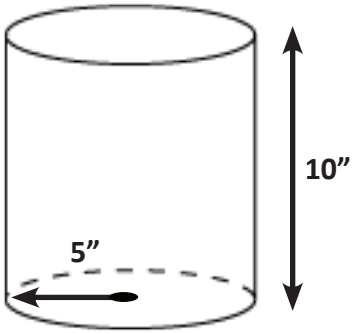


MATH AT WORK: *SURFACE AREA AND VOLUME OF A CYLINDER*

Now practice finding the surface area and volume of a cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r stands for the radius of the cylinder and *h* stands for its height. The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$.

Here is an example: A cylinder has a radius of 5 inches and a height of 10 inches. To find the volume, use the formula $V = \pi r^2 h$. Substitute 5 for r and 10 for h. Therefore $V = \pi \times 5 \times 5 \times 10 = 250\pi$ cubic inches.

To find the surface area of the cylinder, use the formula $SA = 2\pi r^2 + 2\pi rh$. Again substitute 5 for r and 10 for h. Therefore, $SA = 2\pi \times 5 \times 5 + 2\pi \times 5 \times 10 = 50\pi + 100\pi = 150\pi$ square inches.



CHECK YOUR UNDERSTANDING

If a cylinder has radius of 14” and a height of 3”, figure out what the volume and the surface area are.

Write your answers to the right. You can use a calculator.

V = _____

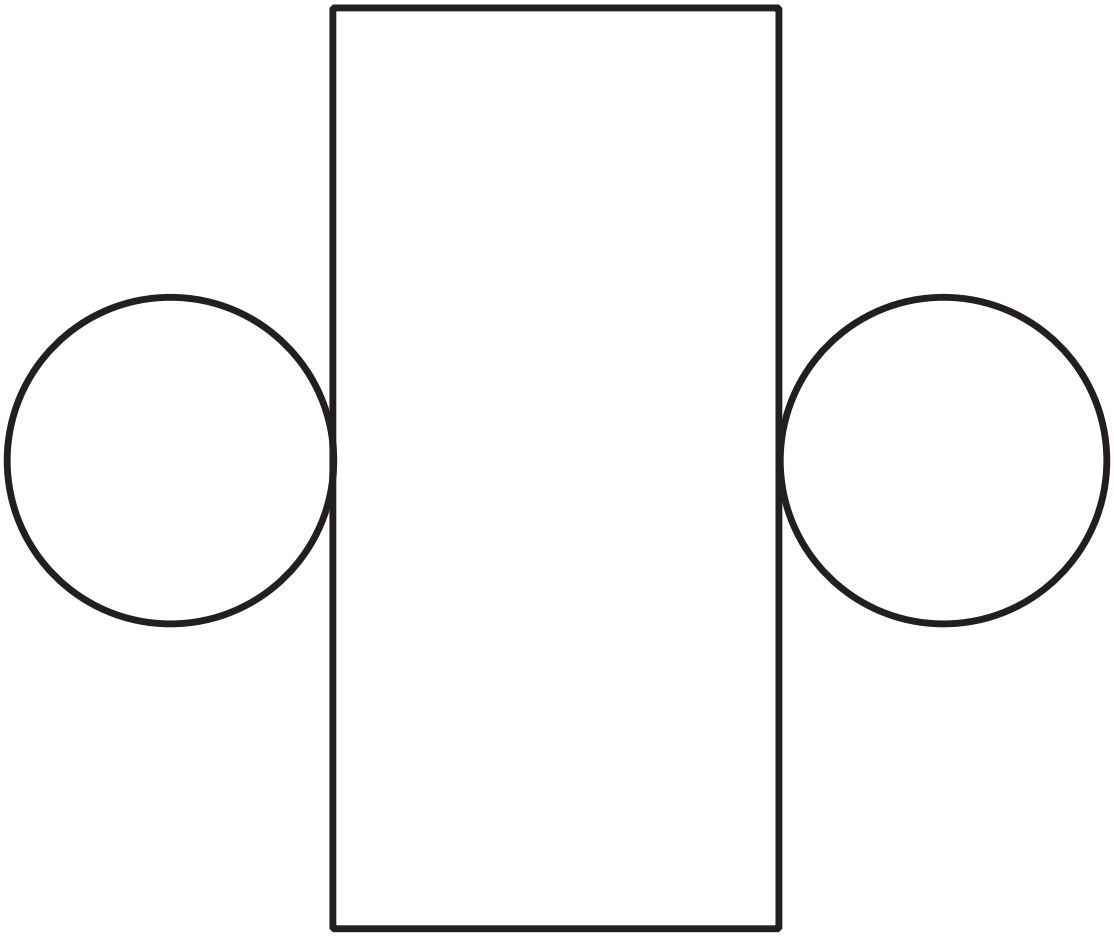
SA = _____

FOLDING SHAPE NETS: *NET OF A CYLINDER*

This diagram shows you what a cylinder looks like when it's cut and stretched out.

Can you visualize that when the rectangle is rolled up from bottom to top, it will reform the cylinder?

The circles will close off the cylinder's top and the bottom.

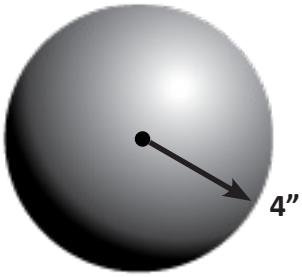


MATH AT WORK: *SURFACE AREA AND VOLUME OF A SPHERE AND HEMISPHERE*

Now practice finding the surface area and volume of a sphere and a hemisphere. First, let's do the sphere. The formula for the volume of a sphere is $V= 4/3\ \pi\ r^3$, where r stands for the radius of the sphere. The formula for the surface area of a sphere is $SA = 4\ \pi\ r^2$.

Here is an example: A sphere has a radius of 4 inches.
To find the volume, use the formula $V= 4/3\ \pi\ r^3$.
Substitute 4 for r.
Therefore $V= 4/3\ \pi\ x\ 4\ x\ 4 = 21.33\ \pi$ cubic inches.

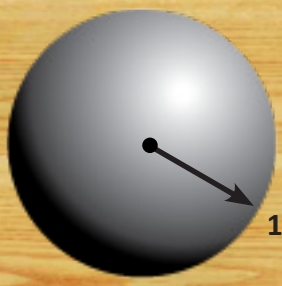
To find the surface area of the sphere, use the formula $SA = SA = 4\ \pi\ r^2$. Again substitute 4 for r.
Therefore, $SA = 4\ \pi\ x\ 4\ x\ 4 = 64\ \pi$ square inches.



CHECK YOUR UNDERSTANDING

If a sphere has a radius of 12”, figure out what the volume and the surface area are.

Write your answers below.
You can use a calculator.

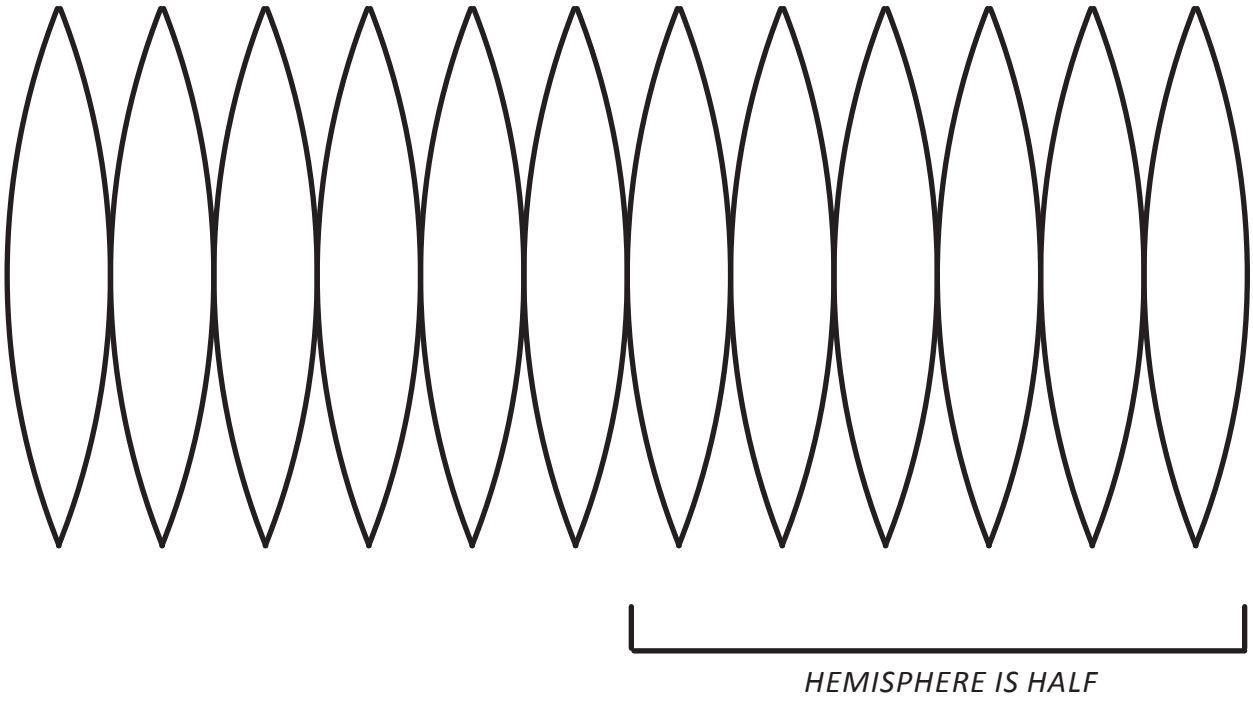


V = _____ **SA =** _____

FOLDING SHAPE NETS: *NET OF A SPHERE*

This diagram shows you what a sphere looks like when it is cut up and stretched out.

Can you visualize that if these shapes were bent around so that the last shape touched the first, this would approximate a sphere?



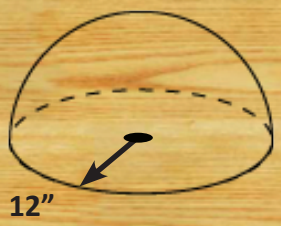
MATH AT WORK: *SURFACE AREA AND VOLUME OF A SPHERE AND HEMISPHERE (CONT.)*

Now think about how you'd calculate the volume and the surface area of a hemisphere. The volume is simply half of the sphere's volume. Therefore, the formula for the volume of a *hemisphere* (half a sphere) is SA= 2/3 π r^3.

CHECK YOUR UNDERSTANDING

If a hemisphere has a radius of 12", figure out what the volume is.

Write your answer below.
You can use a calculator.



V = _____

How would you calculate the surface area of the hemisphere? When you calculate the formula for a hemisphere, it's NOT just half of the SA of a sphere (not just half of 4 π r^2). Remember, we have to add in the area of the circular base.

The surface area of the hemisphere therefore is: **$SA_{hemisphere} = 2 \pi r^2 + \pi r^2 = 3 \pi r^2$**

CHECK YOUR UNDERSTANDING

If a hemisphere has a radius of 12" (same as above) figure out what the surface area is.

Write your answer below. You can use a calculator.

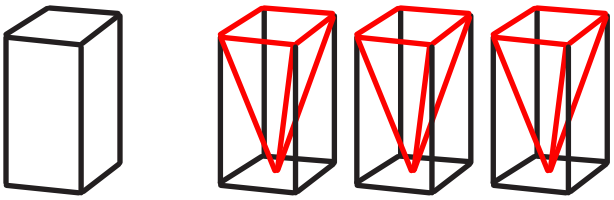
SA = _____

MATH AT WORK: *SURFACE AREA AND VOLUME OF A SQUARE-BASED PYRAMID*

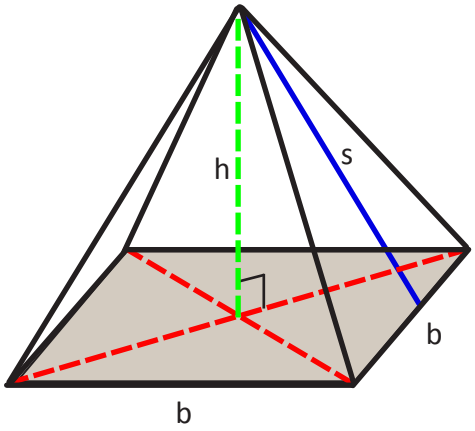
Now practice finding the surface area and volume of a square-based pyramid. The volume of a pyramid is 1/3 the volume of a rectangular prism having the same base and the same height.

The formula for the volume of a pyramid is $V = (1/3)\ b^2h$.

The surface area of the square-based pyramid is $SA = 2bs + b^2$.



Here's an explanation of how we calculate the surface area of a square-based pyramid.
The total surface area = the base area **(b^2)** + the surface area of the four triangular faces.



Notice that this pyramid has two heights:
1. One is the altitude (h)
2. The other is called the "slant height" (s)

Do you see the difference?

- If we know the dimensions of the sides of the base, we can easily figure out the surface area of the base **(b^2)**.
- If we know the slant height (s) of the faces, we can also figure out the area of each of the lateral triangular faces (simply the area of a triangle = ½ bh).
- The surface area, then, equals the surface area of the four triangles (4 x ½ bh or 2 bh) plus the base area (b2).

So, the surface area of the square-based pyramid $=2bs + b^2$.

MATH AT WORK: *SURFACE AREA AND VOLUME OF A SQUARE-BASED PYRAMID (CONT.)*

The formula for the volume of a pyramid is $V = (1/3)\ b^2h$.
The surface area of the square-based pyramid is $SA = 2bs + b^2$.

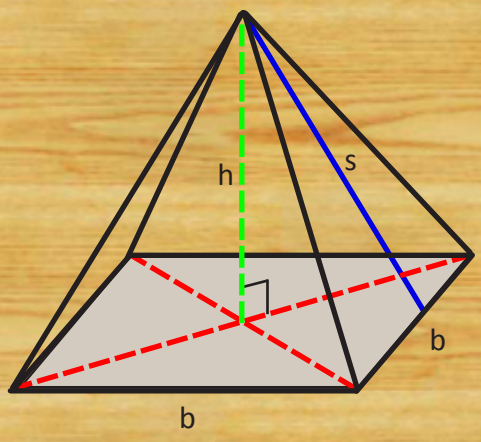
CHECK YOUR UNDERSTANDING

If a square-based pyramid has a height of 24", a base side of 24", and a slant height of 26.83", figure out what the volume and the surface area are.

Write your answers below.
You can use a calculator.

V = _____

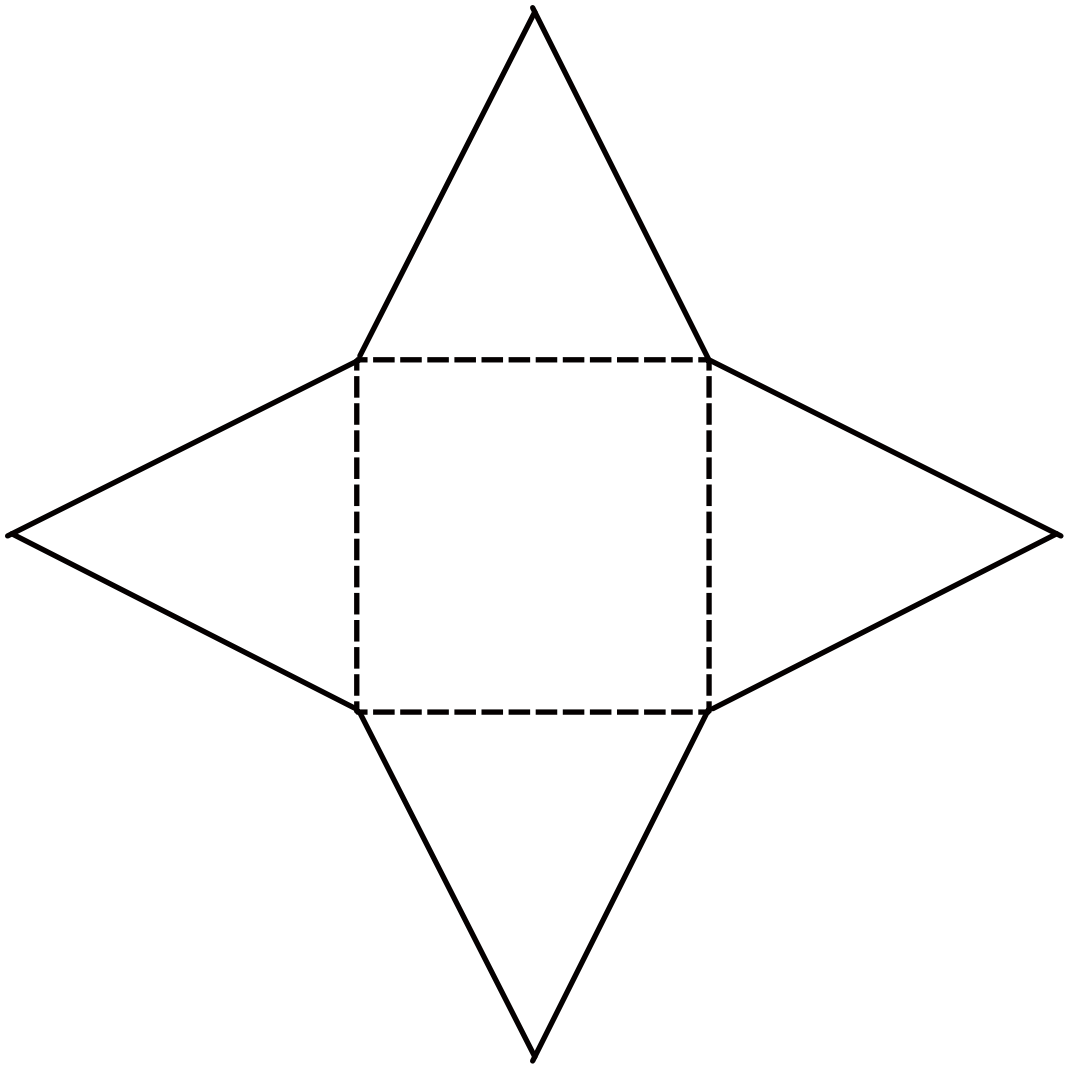
SA = _____



FOLDING SHAPE NETS: *NET OF A SQUARE-BASED PYRAMID*

This diagram shows you what a square-based pyramid looks like when it is cut and stretched out.

Can you visualize that if this shape were folded on the dotted lines, the resulting shape would reform the square-based pyramid?



YOUR CONCLUSIONS

Now that you've investigated possible shelter shapes, what shape do you want to use for your shelter? **(Circle one on the right)**

Explain why your choice is the best shape for your shelter. **Write your response below.**

_____

_____

_____

_____

What is your shelter's volume? (Include units.)
Write your answer below.

_____

What is your shelter's surface area? (Include units.)
Write your answer below.

_____

Please show your work here.

YOUR CONCLUSIONS (CONT.)

Please draw your shelter shape here.

Are there tradeoffs that prompted you to choose the shape you did, other than a different shape that you might have liked even better?

Please write your explanation to your thinking below.

_____

_____

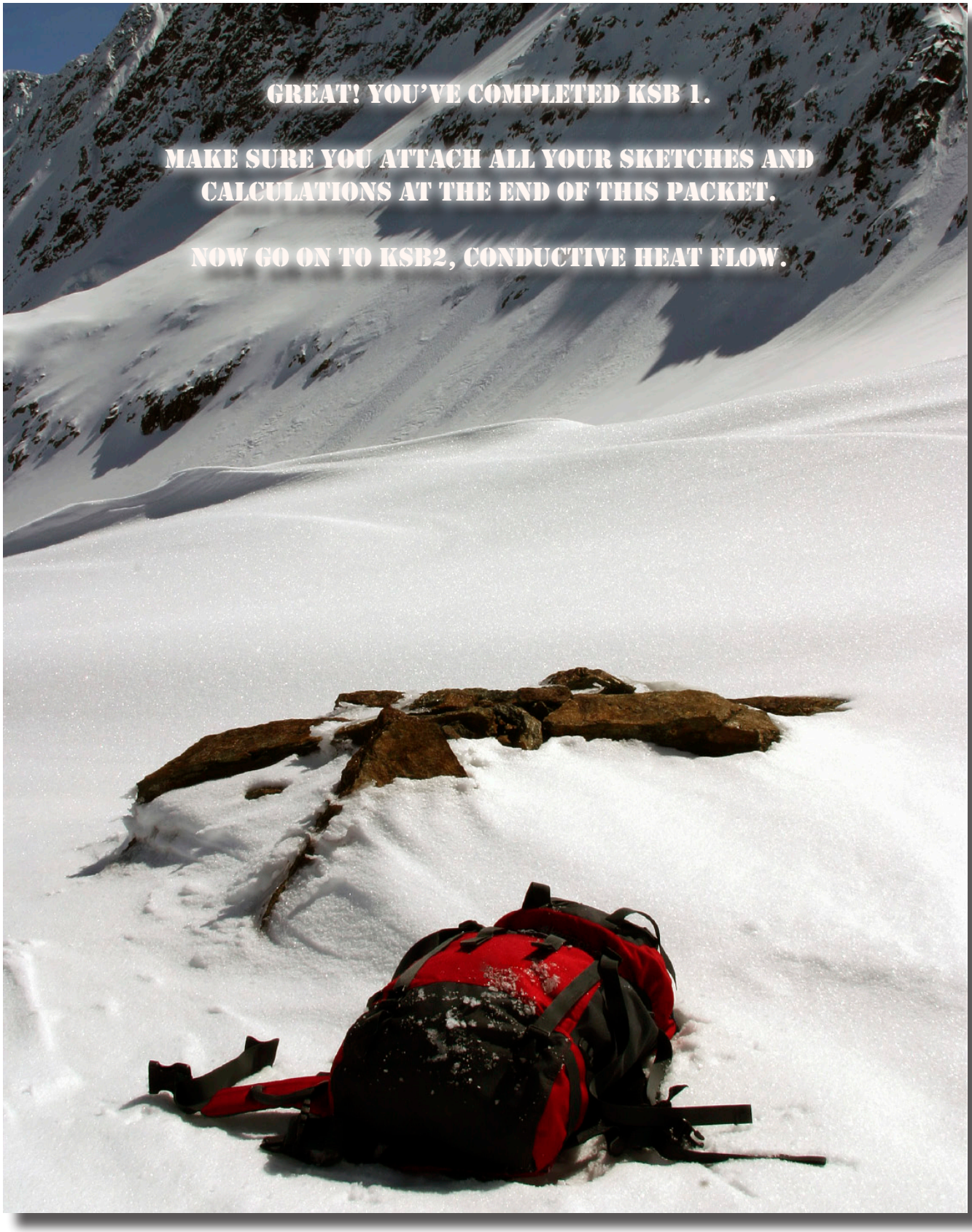
_____

_____

_____

_____

_____


GREAT! YOU'VE COMPLETED KSB 1.
MAKE SURE YOU ATTACH ALL YOUR SKETCHES AND CALCULATIONS AT THE END OF THIS PACKET.
NOW GO ON TO KSB2, CONDUCTIVE HEAT FLOW.