MODULE 1: Nuclear Energy; Exponential Growth and Decay

OVERVIEW

Making predictions about the future is part of what mathematicians and scientists do. Many natural phenomena and physical processes are described by mathematical equations like those describing projectile motion. Mathematics enables us to hit the target we aim at (whether it be an enemy base or the planet Mars). The equations involved are too difficult for the average 6^{th} grader, but if we study the rate at which some simple processes occur, we can come up with models that can be used to predict future behavior; the mathematics required primarily entails multiplication and division with a calculator. Exponential functions in particular are used to describe certain types of growth and decay, and they are applicable to many real-life situations.

One would not expect the 6^{th} graders to be able to solve calculus equations or even understand the symbols with which they are written. The 6^{th} grade teacher can ask students to explore various types of exponential growth and decay, and to use simple graphs to predict what will happen in the future (or, conversely, discover what had happened in the past). Students quickly come to recognize the shape of an exponential curve, and, with a calculator, they can easily verify whether a set of data fits an exponential pattern.

The study of exponential growth and decay can be integrated into a unit on nuclear energy. The strongest force in nature is the nuclear binding force, i.e., the force that holds the nucleus together. It is strong enough to overcome the natural repulsion between the protons in the nucleus, and it can be unleashed in the process known as nuclear fission, or splitting the atom. A chain reaction occurs when enough fissionable material (or "critical mass") is present, and the progress of the reaction is exponential. In a chain reaction, neutrons released in fission produce additional fissions, which in turn produces neutrons, and the process repeats. If each neutron releases two or more neutrons, then the number of fissions doubles in each generation: This is exponential growth. In 10 generations, for example, there are 1024 (or 2^{10} fissions); in 80 generations there are 2^{80} fissions—a number so huge that when written it is approximately equal to 1 followed by 23 zeros.

BRIEF HISTORY

The ancient Greeks had a rudimentary idea about the structure of matter, though they recognized only four "elements": earth, air, fire, and water. The Greek word *atomos* means "indivisible"; all matter could be broken down into its smallest possible particles, which were then further irreducible. The four elements combined in various ways, and in various proportions, to form all matter on earth. Men, for example, were composed mainly of air and fire while women were earth and water. The primitive conceptions of the Greeks foreshadowed the truth, but accurate models of atomic structure did not arrive until the 19^{th} and 20^{th} centuries. Around 1911, Ernest Rutherford discovered the atomic nucleus, and developments in nuclear physics advanced quickly. The Nuclear Age began in 1942, when Enrico Fermi first generated a chain reaction in uranium. The atomic bombs—small by modern standards—that ended World War II in 1945 were a clear demonstration of how much energy could be released by nuclear fission.

INTRODUCTORY ACTIVITIES

1. Paper Folding and Growth.

To introduce exponential growth prior to a discussion of nuclear fission, and to offer a concrete example of how rapidly it can take place, ask the students to fold an ordinary sheet of 8½ × 11 inch paper in half, then in half again, then in half again. The thickness doubles each time. Three folds are fairly simple, and unfolding the paper momentarily will quickly reveal that the thickness is 8 times what it was originally. When I ask the class to guess how many times they can keep on folding the paper in half, their estimates are usually far too high. Folding 10 or 11 times is almost impossible. The thickness of the folded paper has grown exponentially.

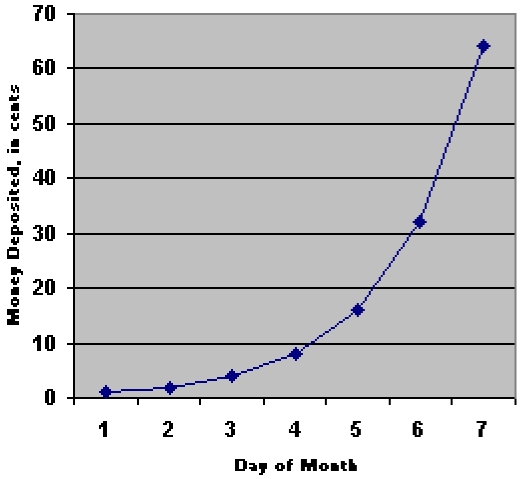
Another simple "experiment" will illustrate the swiftness of exponential growth. The example is an old one, but it will probably be new to most 6th graders. If you put a penny in the bank on the first day of the month, 2 ¢ on the second, 4 ¢ on the third, and so on, doubling the number of cents each day, how much will you deposit on the last day of a 31-day month? First ask for guesses, which are invariably too low. Then have the students calculate day by day, using a chart like the following:

Oct 1 1 ¢ or $.01	**Oct 2** $.02	**Oct 3** $.04	**Oct 4** $.08			
Oct 8 $1.28						
Oct 15 $163.84						
Oct 22						
Oct 29	**Oct 30**	**Oct 31** $10,737,418.24!!!!				

The final figure is truly astonishing!

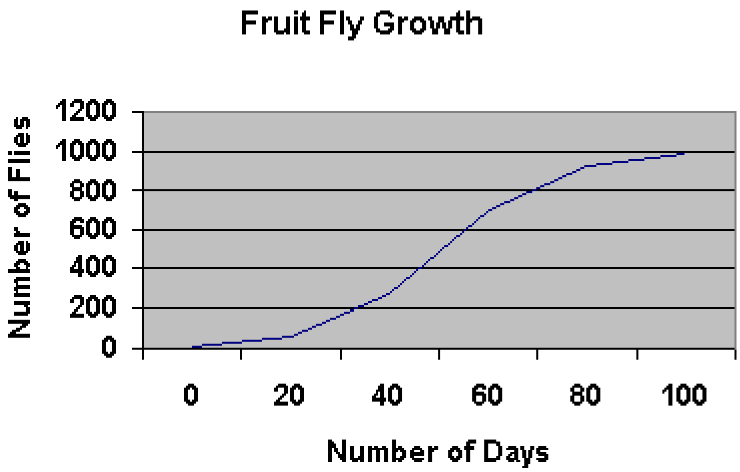
The data for the first 6 days can be used to make a typical exponential graph; the x-axis is labeled with the date; the y-axis is the money deposited on that date, in cents. The line graph, described in simple terms, appears to be a slow starter and then a rapid climber. The shape of the graph is typical of exponential growth.

Figure 1: Exponential Growth



Connections: personal finance (6th graders can play stock market games); computers have changed banking and finance from on-line trading to the use of ATMs; have students seek ways in which technology has affected banking and world economic issues.

The examples that follow deal only with "uninhibited" growth. The teacher should be aware of other types of growth (sometimes called "logistic" growth), where, for example, the increase of a population is limited by environmental factors, or the rate at which an object cools is limited by the temperature of the surrounding medium. A biologist breeding fruit flies might see his colony grow rapidly, then slacken because of limitations imposed by the environment, available food, etc. A graph of fruit fly reproduction in a laboratory might look like the following:



2. World Population Growth

The following table gives the population of the earth in millions of people:

Year	Population (millions)
1986	4936
1987	5023
1988	5111
1989	5201
1990	5329
1991	5422

To determine whether the growth over these six years is exponential, find the *common ratio*, i.e., divide each number in the second column by the number that precedes it.

5023 ÷ 4936 **.** 1.018, 5111 ÷ 5023 **.** 1.018, 5201 ÷ 5023 **.** 1.018. (The symbol "**.** " means "approximately equals.") Since the common ratios are the same, the growth is exponential. That means the table can be extended by multiplying the last entry in the second column by 1.018, and so forth. Have the students use calculators to continue the table and predict the population of the earth in the year 2000.

Year	Population
1992	5422 × 1.018 = 5520
1993	5530 × 1.018 =
1994	
1995	
1996	
1997	
1998	
1999	
2000	

The last entry in the table given above was 5422, for the year 1991. Multiply it by 1.018 to obtain 5530, the projected population for the year 1002. Continue the process.

In fact, the world's population reached 6 billion in 1999, so the model is only a close approximation.

Connections: mathematics (calculator use); social studies and environmental issues; technological advances in farming and medicine support population growth; medical advances allow people to live longer. If growth continues, new resources will have to be developed, in which technology will play a major role (space exploration, undersea mining, etc.).

3. NFL Average Salaries

Year	Salary, × \$1000
1987	220

1988	250
1989	319
1990	365
1991	425
1992	492

Proceed as before: find the common ratio; verify that the growth is exponential. Ask students to extend the table year by year until the average salary is 1000 × $1000, or one million dollars.

Connections: training equipment (technological innovations) and dietary advances lead to bigger, stronger athletes; new equipment leads to new sport and new injuries, i.e., rollerblades; increased incidence of knee and wrist injuries; lighter material and padding in football uniforms results in greater speed and more head injuries; new surgical techniques are developed to deal with new injuries.

4. Coasting to a Stop: An example of exponential decay

The data in the following table give the velocity of a skater who has reached a speed of 7 meters per second and then wishes to coast to a stop. Time is given in 5 second intervals; speed is in meters per second.

Time (seconds)	**Speed (meters/sec)**
0	7
5	5.4516
10	4.2457
15	3.3066
20	2.5752
25	2.0055

The common ratio in this case is less than 1:

5.4516 ÷ 7 = .7788

4.2457 ÷ 5.4516 = .7788

To obtain further entries, multiply the last number in the second column by 0.7788.

Continue until the speed reaches approximately 0.5 m/sec.

Connections: forces and motion; velocity and acceleration; improvements in braking technology lead, ironically, to faster driving and more accidents. Further experiments: Explore the friction of various surfaces (aluminum foil, carpet, waxed paper, sandpaper) to see how they cause a moving object to slow down. Ask students to find further examples of growth data (e.g., inflation) and decay (e.g., depletion of natural resources).

5. Simple Interest: Students Generate Their Own Data

Money deposited in a bank accumulates interest at an exponential rate, if the interest is compounded annually. (It also accumulates interest exponentially if the interest is compounded more frequently, but the mathematics in this case is perhaps too complex for 6^{th} graders.) Suppose you start with $5000 and invest it in an account that pays 4% interest annually. The common ratio for calculating growth is 1.04. Have students complete the following table by multiplying the first entry in the second column by 1.04, then the answer by 1.04 again, and so forth, for a period of 10 years:

Year	**Amount in Bank**
2000	$5000
2001	
2002	
2003	
2004	
2005	
2006	
2007	
2008	
2009	

$5000 \times 1.04 = 5200$. Put this number in the second column.

$5200 \times 1.04 = 5408$.

Continue until column is completed.

At this rate, no one's going to be a millionaire soon!

<u>Connections</u>: familiarize students with money management: have them keep their own checking accounts, make budgets, etc.

NUCLEAR FISSION AND RADIOACTIVE DECAY

Nuclear Fission

We study nuclear fission as part of a long, 6^{th} grade instructional unit on energy. In a nuclear chain reaction, the first neutron splits the uranium-235 nucleus into two fairly heavy pieces (usually barium and krypton) and releases two neutrons. These two neutrons go on to split more nuclei, releasing more neutrons, which split more nuclei, and so on. The number of possible fissions each time is a power of 2, and the table below, with t an arbitrary (and very small) time unit, summarizes the number of nuclei split after each time interval:

Time Interval (*t*)	**Nuclei Split (N)**
1	1
2	2
3	4
4	8
5	16

If students graph the data in the table above, the shape of the curve obtained after plotting the points will be typical of exponential functions. Notice, in particular, how quickly it is growing after only five "generations."

<u>Connections</u>: atomic bombs; fission reactors; alternative energy sources; design project: a nuclear submarine, including all systems needed for safe operation; read the book *20,000 Leagues Under the Sea*. Simulation of nuclear fission can be done with dominoes set up so that when the first is pushed over, it knocks over two more, each of which knock over two more, and so forth. Discuss the difference between nuclear power plants and conventional fuel-burning plants. In each case, the energy released (by fission or by burning fuel) is used to make steam that turns the turbines that generate electricity.

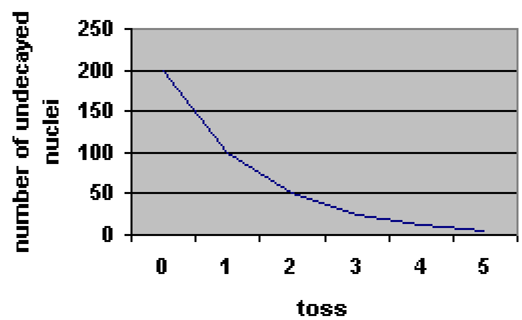
Experiment in Exponential Decay: The Decay of a Radioactive Nucleus

Students are divided into groups, and each group is given 200 chips (or counters or coins) with a different design on each side, perhaps black on one side and white on the other (or heads on one side, tails on the other). The chips are put into a shaker box and then spilled out onto the table. Let the black side (or the tails side) represent the nuclei that have decayed; discard all the chips that turn up black. Count the white ones (or heads) remaining, return them to the shaker, and continue the experiment until almost all the chips are discarded. Enter data in a table like the following:

"Toss"	Number of White Chips
Start	200
1	
2	
3	
4	
5	

Approximately half the number of chips should be discarded each time; don't worry if the actual number is not exactly half.

Graph the data obtained, with "toss" on the horizontal axis and "number of white chips" on the vertical axis. Let "start" = 0 on the graph. The curve should resemble the following:



Each "toss" or shake of the box represents a specific time period. This time period is called the **half-life** of the element. By definition, half-life is the time required for half a sample of a substance to decay. In the next activity, half-lives of various nuclei are given. Students will be asked to predict behavior without necessarily using graphs or charts.

<u>Connections</u>: mathematics (graphing); science (atomic structure, radioactivity, and half-life); environmental issues.

Half-Lives of Certain Radioactive Elements:

The half-lives of various radioactive elements are given below:

Radium 1600 years

Thorium 1.9 years

Uranium-238 4.5 billion years

Bismuth-211 2 minutes

Tritium 12 years

Strontium-90 29 years

Xenon 8 days

Plutonium 24,119 years

Notice the wide variety; especially notice the huge half-lives of the dangerous elements Plutonium, Uranium-238, Radium, and Thorium. All of these can be by-products of fission reactors.

Tritium, an isotope of hydrogen, has a half-life of 12 years. To understand what this means, assume that one has 400 grams of tritium on hand. Every 12 years, half of it decays. Students should be able to continue the following sequence:

Start: 400 g

12 years later 200 g

24 years later 100 g

36 years later 50 g, etc.

Students should complete a similar table for xenon, which has a half-life of 8 days.

Ask students to imagine what would happen if uranium-238 was considered instead of tritium or xenon. Can all the uranium in nature eventually disappear?

Connections: carbon-14 dating in archeology; disposal of radioactive waste and other environmental issues.

Fission Reactors (using the world wide web)

Students should be able to draw diagrams of chain reactions and fission reactors. There are several websites that provide relevant information, including diagrams of reactors and maps of nuclear power plants locations. Direct students to one or more of the following:

http://library.thinkquest.org/

http://www.eren.doe.gov/

http://www.wattwatchers.utep.edu/

www.expage.com/page/np2fission

www.tast-times.com/edge/nuclear.html

www.engr.wisc.edu/groups